Economic Growth and Sustainable Development, NA0167.

Examination, January 2023, suggested answers

1. Consider the following model, which is a variation of the DHSS model in which there is a resource in infinite supply but costly to extract, and competitive markets:

$$
\begin{aligned}
Y &= (A_L L)^{1-\alpha-\beta} K^\alpha (A_R R)^\beta; \\
\dot{A}_L/A_L &= g; \\
\dot{A}_R/A_R &= g_R; \\
\dot{K} &= s(Y-X) - \delta K; \\
C &= (1-s)(Y-X) \\
R &= \phi X.
\end{aligned}
$$

(a) Analyse the model in the following respects:

i. How Y, R, and w_R (the resource price) develop in the long run, assuming balanced growth;

ii. How well these results match global aggregate observations of Y, R, and w_R for resources such as metals and fossil fuels.

The model is not much use for predicting the future development of the global economy, partly because it does not include any of "Solow's three mechanisms", three ways outlined by Solow (1973) in which a resource-dependent economy can adapt to resource scarcity.

(b) Explain briefly how the model can be extended to include each of Solow's mechanisms (separately).

(c) Take one of Solow's mechanisms and explain how your extended model can be used to shed light on policy questions related to sustainability and natural resources or pollution.

(a) You should find that, given balanced growth, both Y and R grow at rate $g + \beta/(1-\alpha-\beta) \cdot g_R$, whereas w_R is always constant (not just on a b.g.p.) because extraction costs are constant. This is a pretty good fit to long-run global aggregate observations, although there are of course plenty of fluctuations in the data which are not predicted by the model. Note also that oil prices have risen somewhat since 1974, due to market power in the oil market. Again, market power is not in our simple model.

(b) Solow's mechanisms are that if R gets scarce, pushing w_R up, (i) firms can boost A_R through investment in R&D, (ii) firms can switch to substitute resources, and boost their productivity through investment in R&D, and (iii) consumers can switch to products of lower resource intensity.

To capture the first two mechanisms we need to dump Cobb–Douglas and switch to (say) nested CES:

$$
\begin{aligned}
&Y = [(A_L L)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}, \\
\text{where}\quad &R = [(A_C C)^\eta + (A_D D)^\eta]^{1/\eta}
\end{aligned}
$$

and C and D are substitutable resources. Note that $\epsilon < 0$ and $\eta > 0$. To capture the third we need alternative Ys which differ in resource intensity, and which consumers can substitute between depend on price and income.

But why would R get scarce? To capture this we would need to add of model of finite (or inhomogeneous) resource stocks.

(c) Lots of options here...

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2. [A]s the earth's supply of particular natural resources nears exhaustion, and as natural resources become more and more valuable, the motive to economize those natural resources should become as strong as the motive to economize labor. The productivity of resources should rise faster than now—it is hard to imagine otherwise. [Solow, *Is the end of the world at hand?*, Challenge, 1973, p47.]

(a) Between 1800 and 1973 the price of primary energy fell greatly compared to the price of labour. Meanwhile, short-run evidence shows that labour and energy are poorly substitutable for one another, i.e. they are strongly complementary in the production function.

i. Write down the profit-maximization problem of a final-good producer buying labour L and energy E on competitive markets, with a CES production function $Y = [(A_L L)^\epsilon + (A_E E)^\epsilon]^{1/\epsilon}$.

ii. Take first-order conditions in the inputs to find an expression for the relative factor shares of the inputs in terms of their relative quantity, and show how this leads to the following result:

$$\frac{w_E E}{w_L L} = \left(\frac{A_E/w_E}{A_L/w_L}\right)^{\epsilon/(1-\epsilon)}.$$

(a)

(i) $\pi = [(A_L L)^\epsilon + (A_E E)^\epsilon]^{1/\epsilon} - w_L L - w_E E$.

(ii) You should find

$$\frac{w_E E}{w_L L} = \left(\frac{A_E E}{A_L L}\right)^\epsilon.$$

There are lots of ways to do the last step. The easiest is probably to start by multiplying both sides by $\left(\frac{w_E E}{w_L L}\right)^{-\epsilon}$.

(iii) Because $\epsilon < 0$, the result shows that when the energy price falls compared to the wage, the factor share of energy falls. This should lead firms to neglect investment in energy efficiency, which should grow more slowly that labour efficiency. The strength of the effect depends on the knowledge production function, in particular the extent to which knowledge stocks grow in isolation from each other. The more they grow in isolation, the bigger the effect.

iii. Use this result to explain why the fall in the energy price might lead labour-augmenting knowledge A_L to grow faster than energy-augmenting knowledge A_E.

iv. Explain why slow growth of A_E would drive up demand for primary energy (for given labour supply).

(b) Discuss evidence about energy-augmenting knowledge growth, using specific examples.

i. Has A_E grown slowly relative to A_L?

ii. How might we explain these observations?

iii. What is the policy relevance of understanding DTC mechanisms?

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3. Over long time periods we have shifted towards energy-intensive goods such as passenger air travel.

(a) How can such shifts help explain the data in Figure 1, if technological change is assumed to be *unbiased* (i.e. both labour productivity and energy productivity grow at equal rates)?

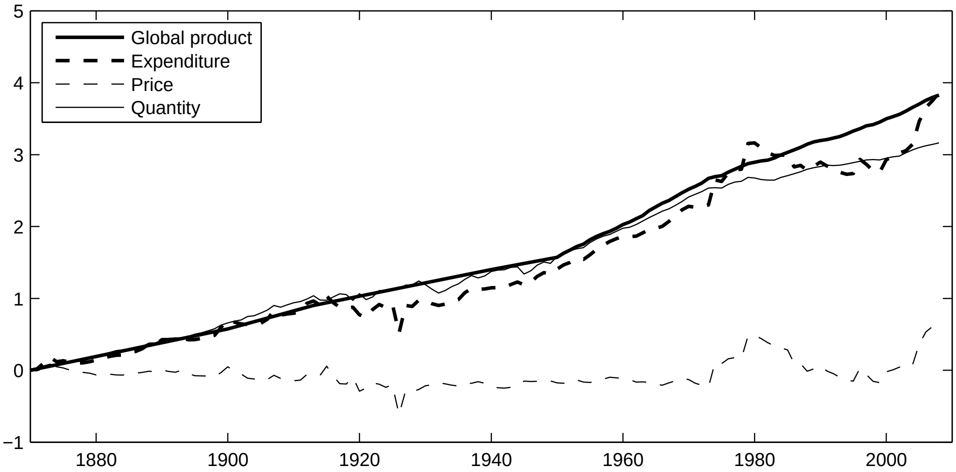

Figure 1: Long-run growth in global production and primary energy* expenditure, price, and quantity. Natural log scale. *Primary energy: Coal, oil, natural gas, and biofuel.

One possible explanation for such shifts is that rich people like energy-intensive stuff. Another is that energy-intensive stuff has got cheaper over time.

(b) Discuss theory and evidence regarding these explanations.

Assume that a regulator wants to reduce energy consumption—and hence carbon emissions—in an energy-intensive sector such as passenger air travel, and is trying to choose between a flight tax and subsidies to energy efficiency research.

(c) What is the relevance of your discussion above to this choice? Do you have other suggestions for the regulator?

(iv) If A_E grows slowly, then as effective labour $A_L L$ increases, more and more effective energy $A_E E$ is required because labour and energy are strongly complementary ($\epsilon < 0$). If A_E doesn't increase then physical inputs E must increase instead.

(b) Use examples such as the generation of light and motive power to argue that A_E has in fact grown rapidly, probably as fast as or faster than A_L. One reason for this is the knowledge stocks (and productivities) do not grow in isolation from one another, they tend to grow together, building on an increasing stock of overall or *general* knowledge. This is important for policy because if knowledge stocks do grow in isolation then we have strong *path dependence*, and the use of research subsidies (for instance to green technologies) can be a very attractive policy option. It might even be the case that we don't need emissions taxes, as suggested by Acemoglu *et al.* 2012, AER.

(a) Start with a simple model in which labour and energy are combined to make final goods, with a low elasticity of substitution between them:

$$Y = [(A_L L)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}.$$

If A_L and A_R grow at the same rate, then we expect R and L to also grow at approximately equal rates, because the inputs are strongly complementary ($\epsilon \ll 0$). But in fact energy quantity has grown much faster than labour supply, approximately tracking GDP. A possible explanation is that there is not a single product Y, but many different products Y_1, Y_2, Y_3, etc. These products differ in their energy intensity, and over time if we shift consumption from less energy-intense products (like education) towards more energy-intense products (like flights) this will push energy use up even though energy efficiency of individual products increases.

(b) If rich people like energy-intensive stuff then we have an *income effect.* Since we are getting richer over time, this could explain the shift. A problem with this idea is that (as shown by Sager (2019)) the energy-intensity of expenditure is more-or-less constant across the expenditure distribution. Furthermore, if anything, those which highest expenditure (the rich) actually have *lower* energy intensity than those with lower expenditures.

If energy-intensive stuff has got cheaper over time, causing the shift, then we have a *substitution effect.* We expect energy-intensive goods to get cheaper because the price of energy has failed to rise. However, the strength of this effect is likely to be modest because the energy share of costs rarely exceeds about 20 percent, even for the most energy-intensive goods such as passenger flight. Another reason such goods may get cheaper is that they are capital intensive, and such goods tend to fall in price relative to services such as education and health care, where productivity increases are slower.

(c) If the substitution effect is strong then there is a risk of *rebound* from increasing energy efficiency. However, again, because of the low energy share in practice this effect is unlikely to be very important. If the income effect is strong and the substitution effect weak then a flight tax may not have much effect, whereas energy efficiency will reduce emissions. A better long-run option would be to tax emissions directly if possible, or to search for new emissions-free technologies for rapid transit.

4. Assume an economy controlled by a social planner with a single final good produced in quantity Y using inputs of labour L and electricity E. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the resource has a small factor share). A_L and L grow exogenously at constant rates. Electricity E is produced using coal X_1, and we choose units such that

$$E = X_1,$$

i.e. the flow of energy is equal to the flow of coal. The extraction cost of coal, w_1, is constant. Furthermore, burning a unit of coal leads to ϕ units of polluting emissions,

$$D = \phi X_1.$$

Utility U is production Y minus total extraction costs, $w_1 X_1$, so

$$U = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D) - w_1 X_1.$$

(a) i. Write down an expression for utility in terms of X_1, and find an expression for $\partial U/\partial X_1$.

ii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very small. (Hint: What does this imply about pollution damages per unit of X_1, compared to extraction costs?)

iii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very large.

iv. Assume that there is an alternative method of producing electricity using an input X_2 that is more expensive ($w_2 > w_1$) but emissions-free. Explain why, as $A_L L$ grows from a very low initial level, the social planner will shift from X_1 to X_2.

(b) Discuss as deeply as you can the relevance of the model to ONE specific real world pollution problem. You should include some or all of the following in your answer:

- What extensions or adaptations we can make to the model so it better fits the specific case in question;
- How the model can help us to understand observations in the specific case;
- What predictions for the future we can make based on the model, in the specific case.

(a) $U = (A_L L)^{1-\alpha} X_1^{\alpha} (1 - \psi\phi X_1) - w_1 X_1$.

$\partial U/\partial X_1 = \alpha Y/X_1 - \psi\phi(A_L L)^{1-\alpha} X_1^{\alpha} - w_1$.

Ignoring damages (which are small in this case) we have $w_1 = \alpha Y/X_1 = \alpha(A_L L/X_1)^{1-\alpha}$. Hence $X_1 = (\alpha/w_1)^{1/(1-\alpha)} A_L L$.

Ignoring extraction costs we instead have $\alpha Y/X_1 = \psi\phi(A_L L)^{1-\alpha} X_1^{\alpha}$, hence

$$X_1 = \frac{\alpha}{1+\alpha}\frac{1}{\psi\phi}.$$

If only X_2 is used then we have $\partial U/\partial X_2 = \alpha Y/X_2 - w_2$.

When $A_L L$ is small, the damage term $\psi\phi(A_L L)^{1-\alpha} X_1^{\alpha}$ is small, so coal is cheapest overall (even allowing for damages) so it is chosen ahead of the clean input. But when $A_L L$ becomes large enough damage costs rise such that $\psi\phi(A_L L)^{1-\alpha} X_1^{\alpha} + w_1 = w_2$ and in an optimally managed economy we will start to switch to the clean input.

(b) Lots of options here ...