Economic Growth and Sustainable Development, NA0167.

Examination, January 2022, suggested answers

1. You are given the following two models.

   - Model 1 (a variation on the 'limits to growth' model).

$$
\begin{aligned}
Y_t &= \min\{A_{Lt}L_t, A_{Rt}R_t\};\\
\dot{A}_L/A_L &= g;\\
\int_0^\infty R_t \mathrm{d}t &\leq S_0.
\end{aligned}
$$

   Labour L is fixed, and hired on perfect markets. The resource R is costless to extract and is of 'open access' character, i.e. no individual or group has property rights over the resource (and it is not storable after extraction). A_R is constant.

   - Model 2 (a variation of the DHSS model with a resource in infinite supply but costly to extract, and competitive markets).

$$
\begin{aligned}
Y &= (A_L L)^{1-\alpha-\beta}K^\alpha (A_R R)^\beta;\\
\dot{A}_L/A_L &= g;\\
\dot{A}_R/A_R &= h;\\
\dot{K} &= s(Y-X) - \delta K;\\
C &= (1-s)(Y-X)\\
R &= \phi X.
\end{aligned}
$$

   (a) i. Consider Model 1, and explain carefully (mathematical reasoning may help) how Y, R, and w_R develop over time, in the long run.

   ii. Consider Model 2, and explain why $w_R = 1/\phi$. Explain also briefly why, on a balanced growth path, Y, K, X, and R must all grow at the same rate. Assume balanced growth and find this rate.

   iii. Discuss briefly the effect of the rate of increase of resource productivity, h, on the rate of increase in resource use $\dot{R}/R$.

   (b) Compare the models in their ability to (i) match and (ii) explain global aggregate observations of GDP growth, and growth rates of resource use and prices for resources such as metals and fossil fuels.

   Now consider coal as the resource. The problem with coal stocks is not that they may run out—causing a crash in production—but that they are too large, leading to climate damages.

   (c) Outline how Model 2 can be adapted in a simple way to explain why, in a growing economy, we first choose to use coal and then—under optimal policy—abandon it, even though extraction costs have not risen. What policies are required according to your adapted model?

(a) In Model 1, both Y and R will grow at rate g until the resource runs out, at which time the economy will collapse; w_R is zero throughout. In Model 2 there is no scarcity, so (assuming perfect markets) prices are equal to unit costs, and resource costs and equal to extraction costs. The extraction input is X, which is just final product, price 1. And each unit of X gives ϕ units of R, so $w_R = 1/\phi$. For balanced growth, fixed proportions of Y must go to C and X, so $\dot{Y}/Y = \dot{X}/X = \dot{R}/R$. Then use the capital accumulation equation to show $\dot{Y}/Y = \dot{K}/K$. Finally differentiate the production function w.r.t. time to show that $\dot{Y}/Y = g + \beta h/(1-\alpha-\beta)$. The higher is h, the faster is growth, and the faster the rate of increase of R. 'Paradoxically', resource efficiency leads to more resource use!

(b) Both models match the long-run trends reasonably well, although according to Model 1 the price of resources is zero rather than constant and positive. In Model 1 resources get sucked in when A_L grows, and there is no way to do without resources at all. This is not realistic. And the idea of open access resources free to extract is obviously wrong. Whereas in Model 2, if resources started to get scarce (which they won't given the assumptions, but will in reality) we would adapt by simply using less, without that having a terrible effect on production, as long as β is low. But how would this work in practice? Would be boost resource efficiency A_R? In the model, this scarcely helps. Would we switch between products? Or find other inputs? Model 2 doesn't have any answers to these questions, so doesn't explain much either.

(c) We need to add alternative resource inputs with different costs and different associated pollution flows, and also add pollution damages which affect production Y. And then we need a regulator who imposes emissions taxes, in the best case equal to the marginal damage costs of pollution flows. For full marks you need to define your suggested model using equations.

2. Assume an economy on an island with a single product, widgets. Widgets are made using labour and energy, in a Cobb–Douglas production function:

$$Y = (A_Y L_Y)^{1-\alpha} E^{\alpha}.$$

The flow of energy inputs E is as follows:

$$E = \left[(A_F R_F)^{\epsilon} + (A_G R_G)^{\epsilon}\right]^{1/\epsilon}$$

where F denotes fossil fuels and G wind power, and $0 < \epsilon < 1$. So energy may be produced using one or both of fossil and wind sources, where the two are good (but not perfect) substitutes; A_F and A_G are productivities, and R_F and R_G are flows of fossil and wind inputs into the energy sector. All markets are perfect, and there is no scarcity.

The flows R_F and R_G are in proportion to flows of widgets X_F and X_G into the mines and windmills respectively, such that

$$R_F = X_F/w_F \quad \text{and} \quad R_G = X_G/w_G.$$

Assume that w_F and w_G vary exogenously, while A_F and A_G are fixed.

(a) Consider the sector in which competitive energy suppliers buy fossil and wind inputs R_F and R_G, and sell output E. Demonstrate—showing your working clearly—that the factor share of wind relative to fossil, defined as S_{GF}, is as follows:

$$S_{GF} \equiv \frac{w_G R_G}{w_F R_F} = \left(\frac{A_G/w_G}{A_F/w_F}\right)^{\epsilon/(1-\epsilon)}.$$

Furthermore, assume that we have data that shows that when the price of wind inputs halves, the share of wind relative to fossil doubles. What is the appropriate choice of ϵ to match this observation?

Now assume that A_F and A_G are also endogenous, and that they develop (at the aggregate level) according to the following equations, where $Z_{G,t}/Z_{F,t} = S_{GF,t}$, and $Z_{G,t} + Z_{F,t} = 10$:

$$A_{F,t+1} = A_{F,t}(1 - \delta + \phi_F Z_{F,t});$$
$$A_{G,t+1} = A_{G,t}(1 - \delta + \phi_G Z_{G,t}).$$

Parameter values: $\delta = 0.01$, $\phi_F = 0.005$, $\phi_G = 0.02$, and $\epsilon = 0.5$. At $t = 0$ we have $A_F = A_G = w_F = 1$, and $w_G = 4$.

(b) i. Show that $S_{GF} = 1/4$, and hence show that there is balanced growth in the energy sector if w_G/w_F remains constant.

ii. Explain (ideally with the help of some simple calculations) what will happen over time if the (exogenous) price of wind inputs w_G halves.

(c) Explain why this model with directed technological change does not match historical experience regarding energy or natural-resource transitions. What changes to the model could help it match historical data better?

(a) Set up the profit-maximization problem of the energy producer and take FOCs to show that

$$\frac{w_G R_G}{w_F R_F} = \left(\frac{A_G R_G}{A_F R_F}\right)^{\epsilon}.$$

Then multiply both sides by

$$\left(\frac{w_G R_G}{w_F R_F}\right)^{-\epsilon}$$

to obtain the result. The appropriate value of ϵ is 0.5.

(b) You should find that both A_F and A_G grow by 3 percent per period, which leaves the factor shares unchanged. Hence as long as relative prices and the parameters remain unchanged, this (balanced) growth will continue. But if w_G falls then the share of wind will increase, leading to more investment in wind technology, and an increase in A_G/A_F. Hence the share of wind will increase further, and so on. The economy will rapidly move to a corner in which only wind power is used. This is the intuition, you should ideally show some calculations.

(c) In reality we see that when an input gets cheaper relative to substitutes, its share increases, but it does not then continue to increase and take over completely. So the DTC mechanism in the model of part (b) is too strong. We need to make productivity in a sector a function not just of investment and prior productivity in that sector, but also of overall productivity in the economy, e.g.

$$A_{F,t+1} = A_{F,t}(1-\delta) + \phi_F Z_{F,t}[H(A_{F,t}, A_t)];$$
$$A_{G,t+1} = A_{G,t}(1-\delta) + \phi_G Z_{G,t}[H(A_{G,t}), A_t],$$

where H is some function.

3. Discuss the following statement.

> Changing consumption patterns are the cause of the rapid growth of global primary energy use illustrated in Figure 1. This implies that rebound effects are very powerful, hence increases in energy efficiency will not on their own reduce energy consumption.

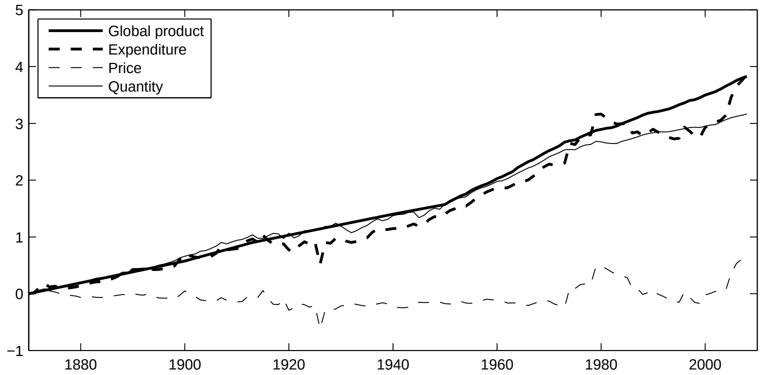

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[1]

This question should ideally be tackled in three stages: (i) What is the cause of the rapid growth? (ii) Does this imply that rebound effects are very powerful? (iii) What are the implications for the benefits of energy efficiency. Here I give some hints about what you could take up.

Broadly, it seems to be true that the rapid growth in energy consumption—tracking global product up to 1974—is indeed largely due to changing consumption patterns. An important alterative explanation would be a lack of energy-augmenting technological progress, but we know that such progress has in fact been rapid.

It is not true that this implies very powerful rebound, since the changes could be caused either by substitution effects (which are strongly linked to rebound) or income effects (which are not).

To the extent that income effects are the cause (and it seems likely that they are an important cause) increases in energy efficiency do help to keep energy use down; without the historical improvements we have observed, the global increases in energy consumption would have been even larger.

---

[1] Energy: Coal, oil, natural gas, and biofuel.

4. Assume an economy controlled by a social planner with a single final good produced in quantity Y using inputs of labour L and electricity E. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} E^{\alpha}(1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the resource has a small factor share). A_L and L grow exogenously at constant rates. Electricity E is produced using coal X_1, and we choose units such that

$$E = X_1,$$

i.e. the flow of energy is equal to the flow of coal. The extraction cost of coal, w_1, is constant. Furthermore, burning a unit of coal leads to ϕ units of polluting emissions,

$$D = \phi X_1.$$

Utility U is production Y minus total extraction costs, $w_1 X_1$, so

$$U = (A_L L)^{1-\alpha} E^{\alpha}(1 - \psi D) - w_1 X_1.$$

(a)
   i. Write down an expression for utility in terms of X_1, and find an expression for $\partial U/\partial X_1$.
   ii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very small. (Hint: What does this imply about pollution damages per unit of X_1, compared to extraction costs?)
   iii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very large.
   iv. Describe the development of the economy over time assuming that at $t = 0$, $A_L L$ is very small.

(b) Assume that there is an alternative method of producing electricity using an input X_2 that is more expensive ($w_2 > w_1$) but emissions-free. Find an expression for $\partial U/\partial X_2$ assuming that only X_2 is used, and (by comparing this expression to the one you derived earlier for $\partial U/\partial X_1$) explain why, as $A_L L$ grows, the social planner will shift from X_1 to X_2.

(c) The model as stated concerns a single global economy (or an isolated country without trade or other interactions). Discuss what an extended model with many countries at different stages of development might predict with regard to *either* (i) local pollutants such as air pollution in cities, *or* (ii) global pollutants such as CFCs and CO_2. Relate the predictions to real-world observations.

(a) $U = (A_L L)^{1-\alpha} X_1^{\alpha}(1 - \psi\phi X_1) - w_1 X_1$.

$\partial U/\partial X_1 = \alpha Y/X_1 - \psi\phi(A_L L)^{1-\alpha} X_1^{\alpha} - w_1$.

Ignoring damages (which are small in this case) we have $w_1 = \alpha Y/X_1 = \alpha(A_L L/X_1)^{1-\alpha}$. Hence $X_1 = (\alpha/w_1)^{1/(1-\alpha)} A_L L$.

Ignoring extraction costs we instead have $\alpha Y/X_1 = \psi\phi(A_L L)^{1-\alpha} X_1^{\alpha}$, hence

$$X_1 = \frac{\alpha}{1+\alpha}\frac{1}{\psi\phi}.$$

Over time, emissions initially track growth, then gradually level off. Y grows initially at the growth rate of $A_L L$, and its growth rate slows very slightly when growth in coal inputs stops.

(b) $\partial U/\partial X_1 = \alpha Y/X_2 - w_2$.

When $A_L L$ is small, the damage term $\psi\phi(A_L L)^{1-\alpha} X_1^{\alpha}$ is small, so coal is cheapest overall (even allowing for damages) so it is chosen ahead of the clean input. But when $A_L L$ becomes large enough damage costs rise such that $\psi\phi(A_L L)^{1-\alpha} X_1^{\alpha} + w_1 = w_2$ and in an optimally managed economy we will start to switch to the clean input.

(c) With regard to local pollutants, we should see that countries tend to act on a given local pollutant at approximately the same stage of their development, so the richest countries should move first. However, these 'early movers' may create technology or other spillovers which induce the lower-income countries to act relatively sooner. For instance, when California acted on air pollution in the 1970s, catalytic converters were installed in all cars, which soon spread globally. A side-effect was to hasten the removal of lead from petrol, since lead clogged up the converters and stopped them from working.

Staying with local pollutants, there is no reason to suppose that a given country will clean up all pollutants at the same time; the timing will differ from pollutant to pollutant depending on costs and benefits. This matches well with the evidence, where we see a successive tightening of environmental regulations, with action taken to remove more and more polluting substances from the production process. Recent examples in the richest countries include efforts to remove neonicotinoids from agriculture, and NO_x emissions from diesel cars.

Concerning global pollutants, we should expect to see the richest countries pushing for global reductions ahead of the lower-income countries, and hence pressure being put on the lower-income countries to act. We see this clearly in the historical negotiations over CFCs, and in current negotiations over climate. However, there is plenty of room for different valuations based on other factors, given (for instance) uncertainty over damages. Hence for instance the reluctance in much of the US to act on climate compared to Europe, most likely due to a relative reluctance in the US to believe in high future damages, due to ideological differences compared to Europe.

We also expect to see global free-riding, so problems with negotiations, and action much later/weaker than would be taken in a global optimum. This clearly corresponds to observations.