Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator.

You have 3 hours to write your answers.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. You are given the following two models.

   - Model 1 (a variation on the 'limits to growth' model).

$$\begin{aligned} Y_t &= \min\{A_{Lt}L_t, A_{Rt}R_t\}; \\ \dot{A}_L/A_L &= g; \\ \int_0^\infty R_t \mathrm{d}t &\leq S_0. \end{aligned}$$

     Labour L is fixed, and hired on perfect markets. The resource R is costless to extract and is of 'open access' character, i.e. no individual or group has property rights over the resource (and it is not storable after extraction). A_R is constant.

   - Model 2 (a variation of the DHSS model with a resource in infinite supply but costly to extract, and competitive markets).

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha (A_R R)^\beta; \\ \dot{A}_L/A_L &= g; \\ \dot{A}_R/A_R &= h; \\ \dot{K} &= s(Y - X) - \delta K; \\ C &= (1-s)(Y - X) \\ R &= \phi X. \end{aligned}$$

   (a) i. Consider Model 1, and explain carefully (mathematical reasoning may help) how Y, R, and w_R develop over time, in the long run.
   ii. Consider Model 2, and explain why $w_R = 1/\phi$. Explain also briefly why, on a balanced growth path, Y, K, X, and R must all grow at the same rate. Assume balanced growth and find this rate.
   iii. Comment briefly on the effect of the rate of increase of resource productivity, h, on the rate of increase in resource use $\dot{R}/R$.

   (b) Compare the models in their ability to (i) match and (ii) explain global aggregate observations of GDP growth, and growth rates of resource use and prices for resources such as metals and fossil fuels.

   Now consider coal as the resource. The problem with coal stocks is not that they may run out—perhaps causing a crash in production—but that they are too large, leading to climate damages.

   (c) Outline how Model 2 can be adapted in a simple way to explain why, in a growing economy, we first choose to use coal and then—under optimal policy—abandon it, even though extraction costs have not risen. What policies are required according to your adapted model?

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2. Assume an economy on an island with a single product, widgets. Widgets are made using labour and energy, in a Cobb–Douglas production function:

$$Y = (A_Y L_Y)^{1-\alpha} E^\alpha.$$

   The flow of energy inputs E is as follows:

$$E = [(A_F R_F)^\epsilon + (A_G R_G)^\epsilon]^{1/\epsilon}$$

   where F denotes fossil fuels and G wind power, and $0 < \epsilon < 1$. So energy may be produced using one or both of fossil and wind sources, where the two are good (but not perfect) substitutes; A_F and A_G are productivities, and R_F and R_G are flows of fossil and wind inputs into the energy sector. All markets are perfect, and there is no scarcity.

The flows R_F and R_G are in proportion to flows of widgets X_F and X_G into the mines and windmills respectively, such that

$$R_F = X_F/w_F \quad \text{and} \quad R_G = X_G/w_G.$$

Assume for now that w_F and w_G vary exogenously, while A_F and A_G are fixed.

(a) Consider the sector in which competitive energy suppliers buy fossil and wind inputs R_F and R_G, and sell output E. Demonstrate—showing your working clearly—that the factor share of wind relative to fossil, defined as S_{GF}, is as follows:

$$S_{GF} \equiv \frac{w_G R_G}{w_F R_F} = \left(\frac{A_G/w_G}{A_F/w_F}\right)^{\epsilon/(1-\epsilon)}.$$

Furthermore, assume that we have data that shows that when the price of wind inputs halves, the share of wind relative to fossil doubles. What is the appropriate choice of ϵ to match this observation?

Now assume that A_F and A_G are also endogenous, and that they develop (at the aggregate level) according to the following equations, where $Z_{G,t}/Z_{F,t} = S_{GF,t}$:

$$A_{F,t+1} = A_{F,t}(1 - \delta + \phi_F Z_{F,t});$$
$$A_{G,t+1} = A_{G,t}(1 - \delta + \phi_G Z_{G,t}).$$

Parameter values: $\delta = 0.01$, $\phi_F = 0.005$, $\phi_G = 0.02$, and $\epsilon = 0.5$. At $t = 0$ we have $A_F = A_G = w_F = 1$, and $w_G = 4$. Furthermore, total research inputs $Z_G + Z_F = 10$ for all t.

(b) i. Show that $S_{GF} = 1/4$, and hence show that there is balanced growth in the energy sector if w_G/w_F remains constant.

ii. Explain (ideally with the help of some simple calculations) what will happen over time if the (exogenous) price of wind inputs w_G halves.

(c) Explain why this model with directed technological change does not match historical experience regarding energy or natural-resource transitions. What changes to the model could help it match historical data better?

3. Discuss the following statement.

Changing consumption patterns are the cause of the rapid growth of global primary energy use illustrated in Figure 1. This implies that rebound effects are very powerful, hence increases in energy efficiency will not on their own reduce energy consumption.

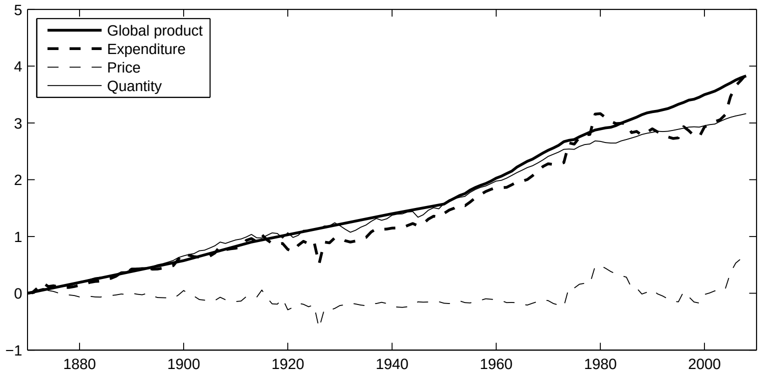

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[1]

[1]Energy: Coal, oil, natural gas, and biofuel.

4. Assume an economy controlled by a social planner with a single final good produced in quantity Y using inputs of labour L and electricity E. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the resource has a small factor share). A_L and L grow exogenously at constant rates. Electricity E is produced using coal X_1, and we choose units such that

$$E = X_1,$$

i.e. the flow of energy is equal to the flow of coal. The extraction cost of coal, w_1, is constant. Furthermore, burning a unit of coal leads to ϕ units of polluting emissions,

$$D = \phi X_1.$$

Utility U is production Y minus total extraction costs, $w_1 X_1$, so

$$U = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D) - w_1 X_1.$$

(a) i. Write down an expression for utility in terms of X_1, and find an expression for $\partial U / \partial X_1$.

ii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very small. (Hint: What does this imply about pollution damages per unit of X_1, compared to extraction costs?)

iii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very large.

iv. Describe the development of the economy over time assuming that at $t = 0$, $A_L L$ is very small.

(b) Assume that there is an alternative method of producing electricity using an input X_2 that is more expensive ($w_2 > w_1$) but emissions-free. Find an expression for $\partial U / \partial X_2$ assuming that only X_2 is used, and (by comparing this expression to the one you derived earlier for $\partial U / \partial X_1$) explain why, as $A_L L$ grows, the social planner will shift from X_1 to X_2.

(c) The model as stated concerns a single global economy (or an isolated country without trade or other interactions). Discuss what an extended model with many countries at different stages of development might predict with regard to *either* (i) local pollutants such as air pollution in cities, *or* (ii) global pollutants such as CFCs and CO_2. Relate the predictions to real-world observations.