Department of Economics
Rob Hart

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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. (a) Consider the DHSS model with competitive markets, zero extraction costs, a fixed saving rate, and constant population:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta, \\ \dot{A}_L/A_L &= g, \\ \dot{K} &= sY - \delta K, \\ C &= (1-s)Y, \\ S &\geq \int_0^\infty R_t \mathrm{d}t. \end{aligned}$$

The market interest rate is constant, denoted ρ.

i. Explain intuitively how the resource price grows in this model economy, based on supply considerations.

ii. Set up the representative final-good producer's problem, take the first-order condition in R, and use the result (plus part (i) above) to find an equation linking $\dot{R}/R$ to $\dot{Y}/Y$ and ρ.

(b) The above results do not match observations for non-renewable natural resource prices and quantities over the last 200 years.

Explain the difference, and suggest alternative assumptions about the resource stock and extraction costs which, when integrated into the above model, yield results which better match the data. Specify your assumptions as exactly as you can, e.g. with the help of a flow diagram of the economy, or equations, or both.

(c) Your new model should fit historical observations regarding non-renewable natural resource prices and quantities, a strength. What are the model's weaknesses, and how might they be addressed in an extended version?

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2. Assume an economy on an island with a single product, widgets. Widgets are made using labour plus either coal or gas, quantities C and D (in tons/year), which are then converted into energy E (in KJ/year):

$$E = A_C C + A_D D.$$

The production function for widgets is Leontief,

$$Y = \min\{A_L L, E\}.$$

Coal and gas are extracted using final goods (widgets) with fixed productivity, set to 1:

$$C = X_C \quad \text{and} \quad D = X_D.$$

A_L, A_C, and A_D are productivities, X_C and X_D are quantities of widgets sent to the energy sector, and L is labour. All markets are perfect, and there is no scarcity. Normalizing the price of widgets to 1 SEK, the prices of coal and gas are thus 1 SEK/ton.

(a) i. Find the condition for coal to be used rather than gas.
ii. Assume that coal is used exclusively, and find an expression for expenditure on coal as a fraction of the value of production Y.
iii. How much is spent on labour inputs?

Now assume that in addition to labour L there is a fixed number of researchers Z, who can be assigned to raising the productivity levels in the economy. For each productivity A_i, the effect of research effort is as follows:

$$A_i(t+1) = A_i(t)(0.999 + \phi Z_i(t)).$$

Furthermore, assume that $A_L = 1$, $A_C = 20$, and $A_D = 5$, while $Z = 20$, $\phi = 0.001$. Finally, assume that researchers are allocated 'myopically' according to current factor shares.

(b) What is the market allocation of researchers between Z_L, Z_C, and Z_D? How will the economy evolve (growth rates of GDP and resource use, factor shares of labour and the resources)?

The government discovers that coal burning is having severe negative effects on the quality of the environment, whereas gas would have no such effects. A pigovian tax (equal to marginal damages) would be 1 SEK/ton.

(c) Find the market allocation if the pigovian tax is applied. In broad terms, how will the economy evolve?

(d) Assuming that the society is patient (low social discount rate), this allocation will not be socially optimal. Explain why not, and discuss alternative (or additional) policies. Discuss what, if anything, we can learn from the model regarding optimal regulation of CO_2 emissions from the burning of fossil fuels.

3. Assume an economy with competitive markets in which total aggregate production is a function of labour-intensive and energy-intensive production, as follows:

$$Y = Y_1^{\alpha} Y_2^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_1 = A_L L,$$

where A_L is labour-augmenting knowledge and L is labour, which is fixed. The energy-intensive good is produced according to the following production function

$$Y_2 = A_R R,$$

where A_R is energy-augmenting knowledge, and R is the energy flow. Energy is extracted using the final product as an input, and one unit of final product yields one unit of energy. Hence if we normalize the price of the final product to 1, the energy price is also 1.

(a) i. Find the relative shares in total product of Y_1 and Y_2. (That is, find $p_1 Y_1/(p_2 Y_2)$, where p_1 and p_2 are the prices of the two goods Y_1 and Y_2.)

ii. Find total energy use R for a given state of the economy. (This is, when L, A_L, and A_R are all fixed and known.)

iii. Assume that a regulator wants to reduce R, and that she can either boost w_r through a tax, or A_R through a research subsidy. Explain which option she should choose in this economy.

(b) Discuss to what extent the above model is relevant to real economies in which the energy share of the most energy-intensive products is typically only about 15 or 20 percent, rather than 100 percent as in the model.

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4. Solow (1973) described three mechanisms through which economies can adapt to future resource scarcity.

(a) State the three mechanisms, and explain their relevance for understanding how the global economy may adapt given the need to reduce the burning of fossil fuels.

(b) Do policy makers in the global economy really need to understand the mechanisms, since theory tells us that in a simple economy with only one market failure—the failure to price emissions—only one instrument is needed, i.e. a Pigovian emissions tax? Discuss.