Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. Consider the following model, which is a variation of the DHSS model in which there is a resource in infinite supply but costly to extract, and competitive markets:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha (A_R R)^\beta; \\ \dot{A}_L/A_L &= g; \\ \dot{A}_R/A_R &= g_R; \\ \dot{K} &= s(Y-X) - \delta K; \\ C &= (1-s)(Y-X) \\ R &= \phi X. \end{aligned}$$

   (a) Discuss the model in the following three respects:

      i. How Y, R, and w_R (the resource price) develop in the long run, assuming balanced growth;

      ii. How well these results match global aggregate observations of Y, R, and w_R for resources such as metals and fossil fuels;

      iii. The model's ability to *explain* (rather than just match) historical observations.

   (b) The model is not much use for predicting the future development of the global economy, partly because it does not include any of "Solow's three mechanisms", three ways outlined by Solow (1973) in which a resource-dependent economy can adapt to resource scarcity.

   Discuss how it might be extended to include each of these mechanisms, while also assuming a more realistic (but still very simple) model of resource stocks.

   (c) Finally, discuss the relative importance of each of the three extensions. Give one example for each extension of how it can add to our understanding of long-run economic development, and appropriate economic policy.

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2. Assume an economy on an island with a single product, hammers. The production function is CES, with inputs of labour L_Y and iron R, with factor-augmenting technology levels A_L and A_R.[1] It can be written

$$Y = [(A_L L_Y)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}.$$

The price of hiring labour is normalized to A_L (so $w_L = A_L$). Meanwhile, iron is extracted from infinite homogeneous stocks by firms with the extraction function[2]

$$R = \phi A_L L_R.$$

   (a) i. Set up the representative hammer-producer's static profit-maximization problem, and use it to derive an expression for the relative factor shares of labour and iron in terms of A_L, A_R, L_Y, and R.

      ii. Multiply each side of this expression by $(w_L L/(w_R R))^{-\epsilon}$, and rearrange, in order to obtain an expression for the relative factor shares of labour and iron in terms of A_L, A_R, w_L, and w_R.

[1] Units: hammers wkr^{-1} year^{-1} and hammers ton-of-iron^{-1}.

[2] So total labour L is divided between production and extraction. Later on we add research labour too.

iii. Find the price of iron w_R, which is equal to the unit cost of extraction of iron, and recall that $w_L = A_L$. Substitute in to your answer to part (ii) above, to obtain an expression for the relative factor shares in terms of A_R and parameters ϕ and ϵ.

iv. Assume that $\phi = 81$ and $\epsilon = -1$, while $A_L = 1$, $A_R = 1$, and $L_Y = 100$. You should find that factor share of labour is 90 percent. Find R, iron extraction. And find Y, hammer production.

(b) Assume that 10 islanders work in the research sector, and that the islanders' knowledge production functions are as follows, where z_l and z_r are measures of research effort (in researcher–years) on labour-augmenting and iron-augmenting knowledge:

$$A_{Lt+1} = A_{Lt}(0.998 + 0.002 z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(0.998 + 0.002 z_{rt+1}).$$

Furthermore, assume that relative investments z_l/z_r in period $t+1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t. (So you do not need to set up and solve the full dynamic problem.)

i. What are relative factor shares in year $t+1$? And by how much (in percent) does R, iron extraction, grow, assuming L_Y is unchanged (i.e. $L_Y = 100$)?

ii. Characterize the development of the economy in the long run.

iii. Explain what happens if, instead, iron starts to run out and its price starts to increase at a constant rate. Comment briefly.

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3. Figure 1 shows how primary energy use has, historically, grown almost as fast as global product, implying that the value of what we produce per unit of primary energy inputs has scarcely increased.

   Discuss critically possible reasons for this observation, including the idea that it is linked to directed technological change, and the idea that it is linked to changing consumption patterns. Discuss policy implications, focusing on the explanation that you judge to be most important.

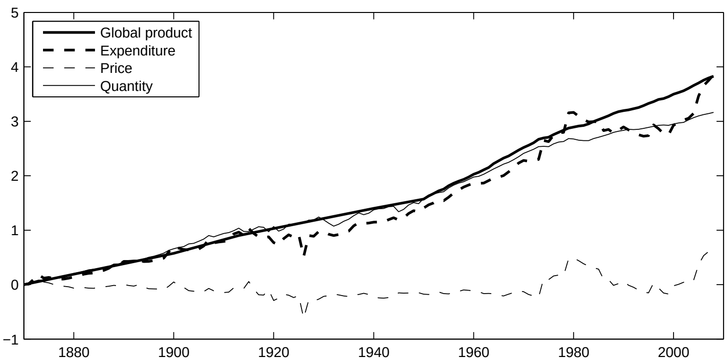

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[3]

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[3]Energy: Coal, oil, natural gas, and biofuel. For data sources see Economic Growth and Sustainable Development, Hart (2017).

4. Assume an economy with competitive markets in which total aggregate production is a function of labour-intensive and energy-intensive production, as follows:

$$Y = Y_1^{\alpha} Y_2^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_1 = a_l L,$$

where a_l is labour-augmenting knowledge and L is labour, which is fixed. The energy-intensive good is produced according to the following production function

$$Y_2 = a_r R,$$

where a_r is energy-augmenting knowledge, and R is the energy flow. Energy is extracted using the final product as an input, and one unit of final product yields one unit of energy. Hence if we normalize the price of the final product to 1, the energy price is also 1.

(a) i. Find the relative shares in total product of Y_1 and Y_2. (That is, find $p_1 Y_1 / (p_2 Y_2)$, where p_1 and p_2 are the prices of the two goods Y_1 and Y_2.)

ii. Find total energy use R for a given state of the economy. (This is, when L, a_l, and a_r are all fixed and known.)

iii. Assume that a regulator wants to reduce R, and that she can either boost w_r through a tax, or a_r through a research subsidy. Explain which option she should choose in this economy.

(b) Discuss to what extent the above model is relevant to real economies in which the energy share of the most energy-intensive products is typically only about 15 or 20 percent, rather than 100 percent as in the model.