Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and non-programable calculator.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. Consider the neoclassical growth model with zero population growth, and add the need for a resource input in the production function. The flow of the resource input is denoted R, there is a total stock of the resource S, and there is no substitute for the resource. There is exogenous labour-augmenting technological progress at a constant rate. The discount rate (interest rate) is exogenously fixed at ρ.

$$\begin{aligned} Y &= (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta}, \\ \dot{A}/A &= g_A, \\ \dot{K} &= sY - \delta K, \\ S &\geq \int_0^{\infty} R_t \mathrm{d}t. \end{aligned}$$

(a) i. Assume that there exists a balanced growth path (b.g.p.) on which all variables grow at constant rates. Find the growth rate of Y on this b.g.p. in terms of g_A and $\dot{R}/R$ (and parameters).

ii. Consider a competitive final-good producer who makes profits $\pi = (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta} - w_l L - w_k K - w_r R$. (The price of Y is normalized to 1.) Take the first-order condition in R to find an expression for w_r in terms of Y and R.

(b) i. Use your previous answers to help you find an expression for the growth rate of w_r in terms of g_A and $\dot{R}/R$.

ii. The resource is free to extract and traded on a perfect market (there are many small suppliers). What must be the growth rate of w_r? Find the growth rate of R in terms of g_A, ρ, α, and β.

(c) Discuss the extent to which this model might help us to understand and predict the long-run development of the global economy.

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2. Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour L and trees R, with factor-augmenting technology levels A_L and A_R. It can be written

$$Y = [(A_L L)^{\epsilon} + (A_R R)^{\epsilon}]^{1/\epsilon}.$$

The parameter $\epsilon = -1$. There are 11 people on the island, of whom 10 work in production and 1 in research, and 20 trees/year wash up on the shore. All markets are perfect.

(a) Assume that—in year t—the islanders have a technology called 'saws' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $A_L = 2$, $A_R = 4$. Calculate total GDP, and the relative factor shares of labour and trees.

(b) Assume that the islanders' knowledge production functions are as follows, where z_l and z_r are the proportions of the year that the single researcher spends working on labour-augmenting and tree-augmenting knowledge:

$$A_{Lt+1} = A_{Lt}(1 + 0.1z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(1 + 0.4z_{rt+1}).$$

Furthermore, assume that relative investments z_l/z_r in period $t + 1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t.

i. What are GDP and relative factor shares in year $t + 1$, if the flow of trees remains constant?

ii. Describe what happens in the short and the long run if the flow of trees declines, approaching zero asymptotically over time.

(c) In the model, directed technological change can solve the problem of combining economic growth with declining resource use. Explain why the specification of the knowledge production functions is key in delivering this very optimistic result. Are these production functions realistic?

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3. Discuss the following statement.

Changing consumption patterns are the cause of the rapid growth of global primary energy use illustrated in Figure 1. This implies that rebound effects are very powerful, hence increases in energy efficiency will not on their own reduce energy consumption.

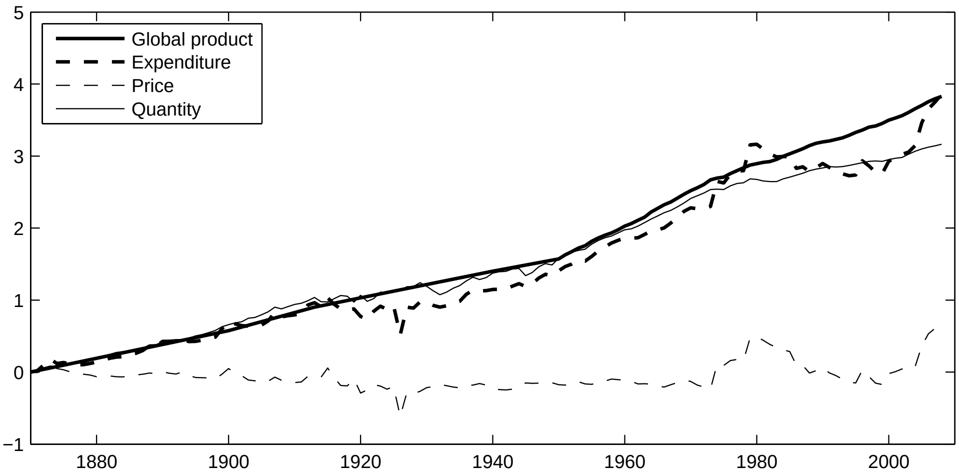

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[1]

[1]Energy: Coal, oil, natural gas, and biofuel. For data sources see Economic Growth and Sustainable Development, Hart (2015).

4. Assume an economy in which total aggregate production is a Cobb–Douglas function of augmented labour and a resource-intensive intermediate good, as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha}.$$

Labour productivity A_L grows exogenously at a constant rate, whereas L is constant:

$$\dot{A}_L / A_L = g_A.$$

The resource-intensive intermediate good is produced according to the following production function, where C is a clean input and D is a dirty input:

$$R = C + D.$$

The inputs C and D are available at fixed exogenous prices w_c and w_d, and $w_c > w_d$. All markets are competitive.

(a) i. Find expressions for $w_l L$ and $w_r R$ as functions of Y. What can we say about the shares of R and L?

ii. Describe how R will be produced in equilibrium, and find its price. (Note: You should find that the price is constant.)

iii. Use your answers to (i) and (ii) above to find the growth rate of R as a function of the growth rate of Y.

iv. Use the aggregate production function and your previous results to find the growth rates of Y, R, and D in terms of g_A.

(b) Now assume that the aggregate production function is actually

$$Y = (A_L L)^{1-\alpha} (C + D)^{\alpha} e^{-\psi D},$$

where ψ is a positive parameter and D represents both the use of input D as an input, and the consequent flow of pollution generated. It follows that the social cost of using input D is $w_d + \psi Y$.

i. Describe the development path of the economy if firms are forced to pay the full social costs of using the inputs C and D, assuming that $w_d + \psi Y$ is initially less than w_c.

ii. Discuss the extent to which this model can help us to understand paths of polluting emissions over time that have been observed for air pollutants such as SO_2.