Department of Economics
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# Examination

Economic Growth and Sustainable Development, NA0167.

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**Rules**

*Permitted aids: Pen, paper, and pocket calculator.*

You have 3 hours to write your answers.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. Consider the following model, a variation on the 'limits to growth' model of the early 1970s.

$$Y_t = \min\{A_{Lt}L_t, A_{Rt}R_t\};$$
$$\dot{A}_L/A_L = g;$$
$$\int_0^\infty R_t \mathrm{d}t \leq S_0.$$

Labour $L$ is fixed, and hired on perfect markets. The resource $R$ is costless to extract and is of 'open access' character, i.e. no individual or group has property rights over the resource (and it is not storable after extraction). $A_R$ is constant.

(a) i. Explain carefully (mathematical reasoning may help) how $Y$, $R$, and $w_R$ develop over time in the model economy, in the long run. Assume that the resource remains 'open access' throughout.

ii. Discuss what, if anything, we can learn from this model about future development of the global economy and what policies are necessary to ensure sustainability. To the extent that we cannot learn from it, explain why not; what key mechanisms are missing from the model that are relevant in the real global economy?

(b) Discuss how we can build more sophisticated models of how the global economy may be affected by (and adapt to) resource scarcity or the need to reduce polluting emissions associated with resource use, and what we can learn from such models. Include the DHSS model of the mid-1970s, and more modern approaches.

2. Assume an economy on an island with a single product, widgets. Widgets are made using labour and energy, in a Leontief production function:

$$Y = (A_Y L_Y)^{1-\alpha} E^{\alpha}.$$

Of production $Y$, $X$ is used to make energy inputs and the rest is consumed. The flow of energy inputs $E$ is as follows:

$$E = A_F L_F + A_R L_R,$$

where $F$ denotes fossil fuels and $R$ renewables. So energy may be produced using one or both of fossil and renewable sources, where the two are perfect substitutes. $A_Y$, $A_F$, and $A_R$ are productivities, and $L_Y$, $L_F$ and $L_R$ are quantities of workers in each sector. All markets are perfect, and there is no scarcity. Normalize the price of a widget to 1 SEK.

Now assume that in addition to labour $L$ there are researchers. A fixed number of researchers work on raising $A_Y$, and as a consequence $A_Y(t+1)/A_Y(t) = 1.2$ (one time period is 10 years). Furthermore, there is a fixed number of researchers $Z$ in the energy sector, divided between $Z_F$ and $Z_R$. And

$$A_F(t+1) = 0.95 A_F(t) + \phi Z_F(t)[\sigma A_Y(t) + A_F(t)]$$
$$\text{and} \quad A_R(t+1) = 0.95 A_R(t) + \phi Z_R(t)[\sigma A_Y(t) + A_R(t)].$$

Assume that $A_{Y0} = 100$, $A_{F0} = 10$, and $A_{R0} = 1$, while $Z = 5$, $\phi = 0.01$, and $\sigma = 0.4$. Finally, assume that researchers are allocated 'myopically' according to current factor shares.

(a) i. Find an expression for the price of energy $w_E$ when it is generated from fossil fuels. Your expression should be in terms of $A_F$ and $w_L$ (the wage, which is the same for all workers). Find an equivalent expression when renewables are used.

ii. Explain why fossil fuels will be used exclusively, and why all researchers will be allocated to fossil research.

iii. Show that $A_Y$ and $A_F$ will grow at equal rates.

Note that when $A_Y$ and $A_F$ grow at equal rates we have balanced growth, and $E$, $Y$, and $w_L$ also grow at the same rate, while $w_E$ is constant.

The government discovers that fossil fuel burning is having severe negative effects on the quality of the environment, whereas renewables would have no such effects. Assume that a Pigovian tax (equal to marginal damages) would add $0.1 w_L$ to the price of energy generated from fossil fuels.

(b) Find the market allocation if the Pigovian tax is applied at $t = 0$. In broad terms, how will the economy evolve? (Think about how the Pigovian tax changes over time.)

(c) Assuming that the society is patient (low social discount rate), this allocation will not be socially optimal. Explain why not, and discuss alternative (or additional) policies.

Discuss briefly what we can learn from the model regarding optimal regulation of $CO_2$ emissions from the burning of fossil fuels.

3. You are given the production and instantaneous utility functions in two models which provide alternative explanations of why consumers may shift towards more energy-intensive goods over time.

   - Model 1.
     There are two products $Y_1$ and $Y_2$ produced by labour and energy respectively.

     $$Y_1 = A_L L;$$
     $$Y_2 = A_E E.$$

     Labour $L$ is fixed, and energy is extracted at fixed unit cost. All markets are perfect. Instantaneous utility is a Cobb–Douglas function of the two:

     $$u = Y_1^{1-\alpha} Y_2^{\alpha}.$$

   - Model 2.
     There is an infinite series of products $Y_i$, and the production function for product $i$ is as follows:

     $$Y_i = (1/2^{i-1}) \min\{A_L L_{Yi}, A_E E_i / 2^{i-1}\},$$

     where $A$ is productivity, $L_Y$ is labour in final-good production, $E$ is the energy input, and $A_E$ is fixed. Consumers have lexicographic preferences such that they always prefer to consume the good with the highest $i$ that they can afford, given that they demand a minimum quantity.

   In both models the productivities $A_L$ and $A_E$ each grow at the constant exogenous rate $g$, and the initial factor share of energy is approximately 5 percent.

   (a) Consider Model 1. Show that the factor share of energy is constant, and explain what this implies about the growth rate of energy use given that the energy price is constant. What happens if energy efficiency $A_E$ increases faster than $A_L$?

   (b) Consider Model 2. Explain why, as $A_L$ and $A_E$ grow, consumers shift to more energy-intensive goods. What are the implications for the growth rate of energy use? What happens if energy efficiency $A_E$ increases faster than $A_L$?

   Swedes' spending on international flights rose rapidly between 1980 and 2018 (much more rapidly than GDP). The result was that energy use and carbon emissions from the sector grew rapidly, despite increasing efficiency of airplanes.

   (c) Explain how each of the models above might be able to shed light on these observations, using the terms 'substitution effect' and 'income effect'. Which model do you think comes closest to the truth?

4. Assume an economy in which total aggregate production is a Cobb–Douglas function of augmented labour and a resource-intensive intermediate good, as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha}.$$

Labour productivity $A_L$ grows exogenously at a constant rate, whereas $L$ is constant:

$$\dot{A}_L / A_L = g_A.$$

The resource-intensive intermediate good is produced according to the following production function, where $C$ is a clean input and $D$ is a dirty input:

$$R = C + D.$$

The inputs $C$ and $D$ are available at fixed exogenous prices $w_c$ and $w_d$, and $w_c > w_d$. All markets are competitive.

(a) i. Find expressions for $w_l L$ and $w_r R$ as functions of $Y$. What can we say about the shares of $R$ and $L$?

ii. Describe how $R$ will be produced in equilibrium, and find its price. (Note: You should find that the price is constant.)

iii. Use your answers to (i) and (ii) above to find the growth rate of $R$ as a function of the growth rate of $Y$.

iv. Use the aggregate production function and your previous results to find the growth rates of $Y$, $R$, and $D$ in terms of $g_A$.

(b) Now assume that the aggregate production function is actually

$$Y = (A_L L)^{1-\alpha} (C + D)^{\alpha} e^{-\psi D},$$

where $\psi$ is a positive parameter and $D$ represents both the use of input $D$ as an input, and the consequent flow of pollution generated. It follows that the social cost of using input $D$ is $w_d + \psi Y$.

i. Describe the development path of the economy if firms are forced to pay the full social costs of using the inputs $C$ and $D$, assuming that $w_d + \psi Y$ is initially less than $w_c$.

ii. Discuss the extent to which this model can help us to explain and predict global patterns in the path of emissions of $CO_2$.