

Core lecture 2

The DHSS model and Solow's mechanisms

October 24, 2022

Background

- Limits to growth?
- The 'Limits' model

The original DHSS model

Neoclassical growth and nonrenewable resource supply: three simple cases

Complex cases: limited resources, costly to extract

Background





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Figure 5: Long-run growth in total consumption compared to growth in total global product, for (a) Metals (tons extracted), and (b) Primary energy from combustion (joules burnt). For sources see EGSD.





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In 1972, the Club of Rome (Meadows *et al*) published 'The limits to growth':

If the present growth trends in world population, industrialization, pollution, food production, and resource depletion continue unchanged, the limits to growth on this planet will be reached sometime within the next one hundred years. The most probable result will be a rather sudden and uncontrollable decline in both population and industrial capacity.





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Note that it is not good enough to say that a trend is 'unsustainable' if it can't continue for ever. On that criterion we could extract zero non-renewable resource stocks from the Earth's crust.

We want to know whether our choices today will be condemned in the future. More specifically, we might have a criterion that future people should have at least the level of welfare that we have.

According to Meadows et al, that criterion is unlikely to be satisfied.





The 'Limits' model

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The Limits to Growth is based on a 'black box' computer model and hence how it works is not easy to see. However, see for instance Nordhaus (1973), https://www.jstor.org/stable/2230846?seq=1.

Let's set up a simple 'Limits' model in discrete time, as follows.





The 'Limits' model



A simple 'Limits' model

Assume that the resource stock is finite and 'open access'.

What is the resource price?

Describe carefully what happens over time if L = 1, $A_L(0) = 1$, $\theta = 1$ (periods are 20 years), and S(0) (the stock of resources at the start of period 0) is 10.

As an economist, what would you say is wrong with this model?

The original DHSS model is partly a response to it.

resource supply: three simple cases

model

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- The model
- The Hartwick rule
- Relevance

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The original papers are: Dasgupta and Heal (1974), Solow 1974 (equity, resources), and Stiglitz (1974). However, they are not a particularly easy read!

See also Hotelling's original article.





The model



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Neoclassical growth and nonrenewable resource supply: three simple cases

Complex cases: limited resources, costly to extract



A limited resource, costless to extract (Hotelling/DHSS), no technological progress.

Assume a social planner who releases a resource flow R which declines over time at rate θ . (So $-\dot{R}/R = \theta$.)

Solve for growth rates on the b.g.p.

What can we say about the initial rate of resource consumption?





The model



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Neoclassical growth and nonrenewable resource supply: three simple cases

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A limited resource, costless to extract (Hotelling/DHSS), no technological progress.

Assume perfect markets and symmetric equilibrium. What can we say about the resource price?

Solve for the b.g.p. in the market economy given that the interest rate is equal to the pure rate of time preference.





The Hartwick rule

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The Hartwick rule

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If we invest resource rents in capital, this guarantees sustainability?

Hartwick, 1977. If, through investment in human-made capital, we can maintain constant overall capital stocks even as natural capital approaches zero then we can sustain utility without technological progress.

But we can't do that if human-made capital depreciates. As pointed out by Hartwick himself in the final paragraph of the paper! And human-made capital does depreciate...





Relevance

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Neoclassical growth and nonrenewable resource supply: three simple cases

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The DHSS model is today a distraction which hinders understanding of real economies by those who cling to it.

Assumption of zero extraction cost not realistic, and leads to resource price prediction which is completely contrary to evidence.

Assumption of no depreciation of human-made capital gives excessive role to physical capital accumulation and distracts attention from the key role of *knowledge* accumulation.





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The original DHSS model

Neoclassical growth and nonrenewable resource supply: three simple cases

• Add exogenous knowledge growth

- Land (and 'flow renewables')
- An abundant resource, costly to extract

• A limited resource, costless to extract (Hotelling/DHSS)

• Historical data

Complex cases: limited resources, costly to extract

Neoclassical growth and nonrenewable resource supply: three simple cases





Add exogenous knowledge growth

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A first step towards reality is to add knowledge growth. For simplicity we make it exogenous.

We then test different scenarios with respect to resource supply.





Land (and 'flow renewables')



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Complex cases: limited resources, costly to extract



The economy with a fixed 'flow' resource.

Assume balanced growth. Characterize the b.g.p.

The representative final-good producer must hire labour, capital, and 'land'. What happens to the price of land over time?





An abundant resource, costly to extract

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The economy with an unlimited resource, costly to extract.

Solve for growth rates on a b.g.p.!





A limited resource, costless to extract (Hotelling/DHSS)



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The economy with a limited resource, free to extract.

Assume a social planner who releases a resource flow R which declines over time at rate θ . (So $-\dot{R}/R = \theta$.)

Solve for growth rates on the b.g.p.

What can we say about the initial rate of resource consumption?





A limited resource, costless to extract (Hotelling/DHSS)



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The economy with a limited resource, free to extract.

Assume perfect markets and symmetric equilibrium. What can we say about the resource price?

Solve for the b.g.p. in the market economy given that the interest rate is equal to the pure rate of time preference.





Historical data

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Figure 1: Long-run growth in consumption and prices, compared to growth in global product, for (a) Metals, and (b) Primary energy from combustion. For sources see EGSD.





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• Non-renewable resources in the long run

• An extraction model with inhomogeneous stocks

• Lessons

Complex cases: limited resources, costly to extract





Non-renewable resources in the long run

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Non-renewable resources in the long run

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It turns out that we can capture the good points from the 'three simple cases' above in a single model economy, which moves through the cases over time.

See Hart 2016. Again, not an easy read!

See also Hart and Spiro (2013) for a much easier read, somewhat related.





Non-renewable resources in the long run

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Consider the picture below, and with its help try to identify as many such factors as you can. Furthermore, categorize them according to whether they should make extraction costs rise or fall over time.







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Now assume a *primitive* economy in which A_Y is very small. What happens?

 $Y = (A_Y L_Y)^{1-\alpha} X^{\alpha}$ $A_D = e^D$ $\dot{A}_Y/A_Y = g$ $\dot{A}_X / A_X = g$ 1 $X = L_X A_X / A_D$ L L_X $\dot{D} = X/\phi$ $\dot{A}_D = \frac{\partial A_D}{\partial t} = \frac{\partial A_D}{\partial D} \frac{\partial D}{\partial t} = A_D \dot{D}$ ф $\Rightarrow \dot{A}_D / A_D = X / \phi$



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Now assume a *balanced growth path* on which X is constant. Characterize the path.



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Now assume a *Hotelling economy* in which the resource is running out. Characterize the path.



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Empirical application?

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We can solve the problems related to resource stocks, extraction costs, and predictions about the price.

I.e. we can solve the problems related to resource supply.

But what about resource demand, and the Cobb–Douglas production function?

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Daly (1997):

In the Solow–Stiglitz variant, to make a cake we need not only the cook and his kitchen, but also some non-zero amount of flour, sugar, eggs, etc. This seems a great step forward until we realize that we could make our cake a thousand times bigger with no extra ingredients, if we simply would stir faster and use bigger bowls and ovens.

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How can we save on flour, sugar, and eggs?

- 1. Reduce waste in the production process;
- 2. Make our cake with almond flour and bananas;
- 3. Eat less cake and more other stuff instead.

More generally:

- 1. Resource efficiency;
- 2. Switch resource inputs;
- 3. Change consumption patterns.

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Solow (1973), p. 46–47, discusses the above 'three mechanisms', focusing on how higher resource prices—caused by scarcity—will tend to drive them:

- Use less of the resource (increase resource efficiency) in production of one or more product categories;
- 2. Shift to an alternative (substitute) resource in production of one or more product categories.
- 3. Substitute on the consumption side away from product categories in which the production process is resource-intensive.

The first two of these three mechanisms may be boosted through directed technological change (DTC), developing (or reducing the costs of) new technologies which use less of the resource or which use alternative resources. The final one may be boosted through changing social norms.

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Note that to explain the past using Solow's mechanisms we need to put them into reverse. Then we have three possible explanations for why resource and energy use has increased (tracking GDP):

- 1. resource-efficient technology has not been developed,
- 2. there has not been substitution on the production side to alternative inputs, such as renewables, and
- 3. consumption patterns have not shifted away from resource-intensive goods.

To understand the mechanisms it is useful to look at past data. Then we can apply what we have learnt to predicting the future and designing policy.

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Can we capture these mechanisms using a simple Cobb–Douglas production function?

What changes could we introduce?

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Why the focus on capital substituting for resources in the DHSS model?

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