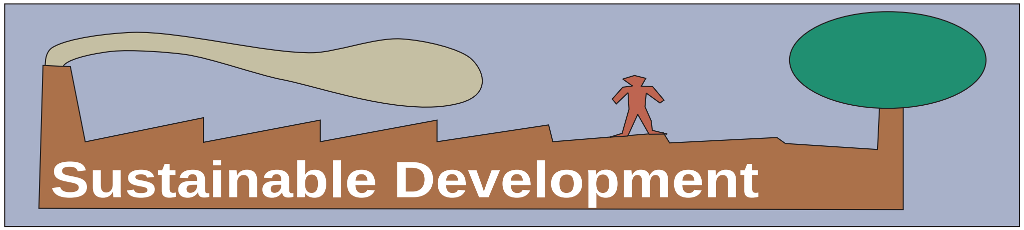
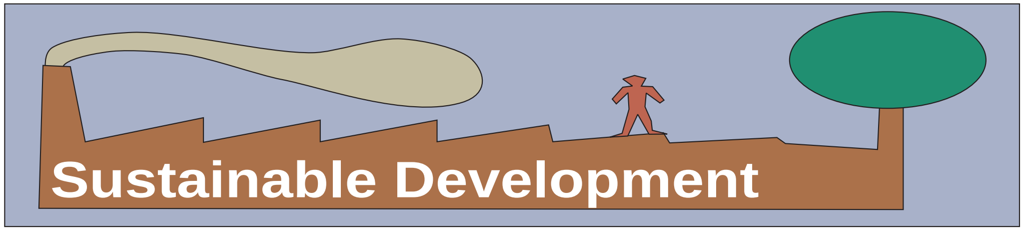




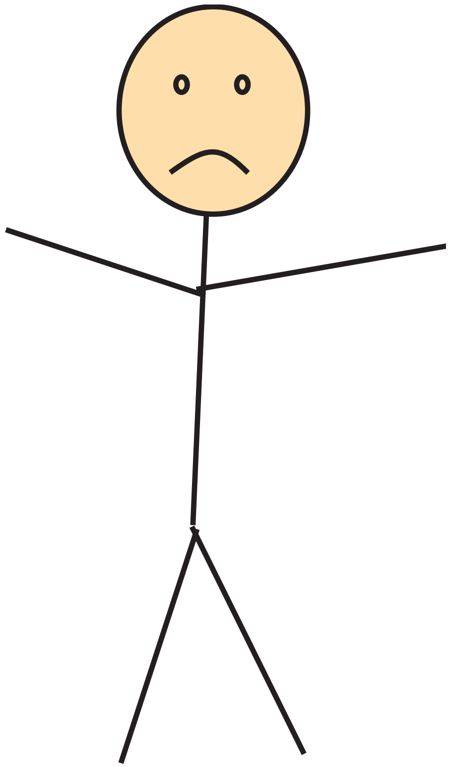
Core lecture 1

Economic growth

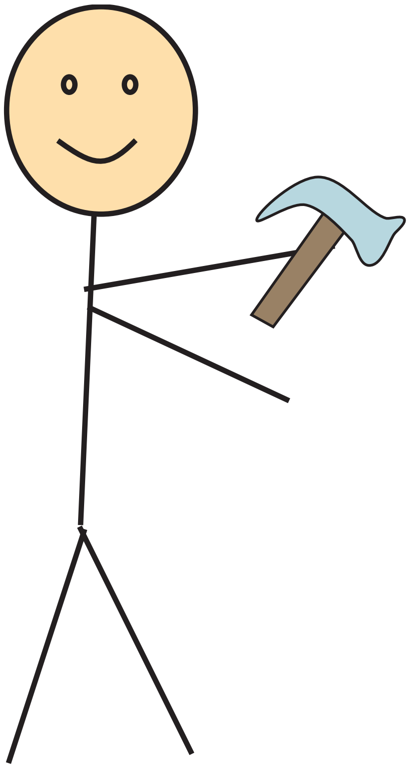
What drives growth?

Accumulation of production factors?

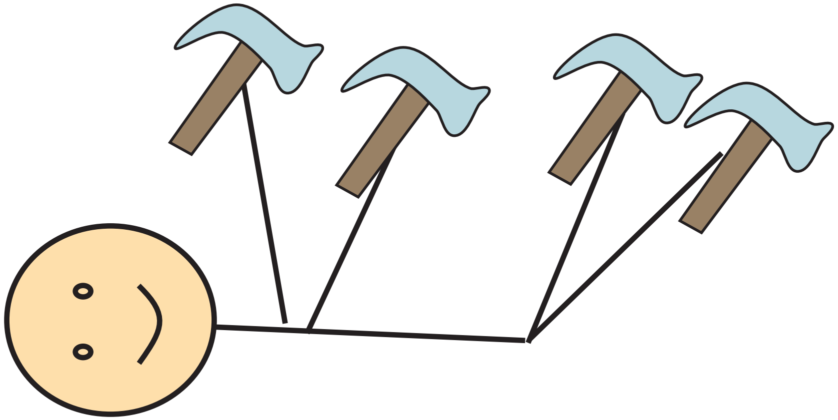
Accumulation of production factors?



Accumulation of production factors?



Accumulation of production factors?



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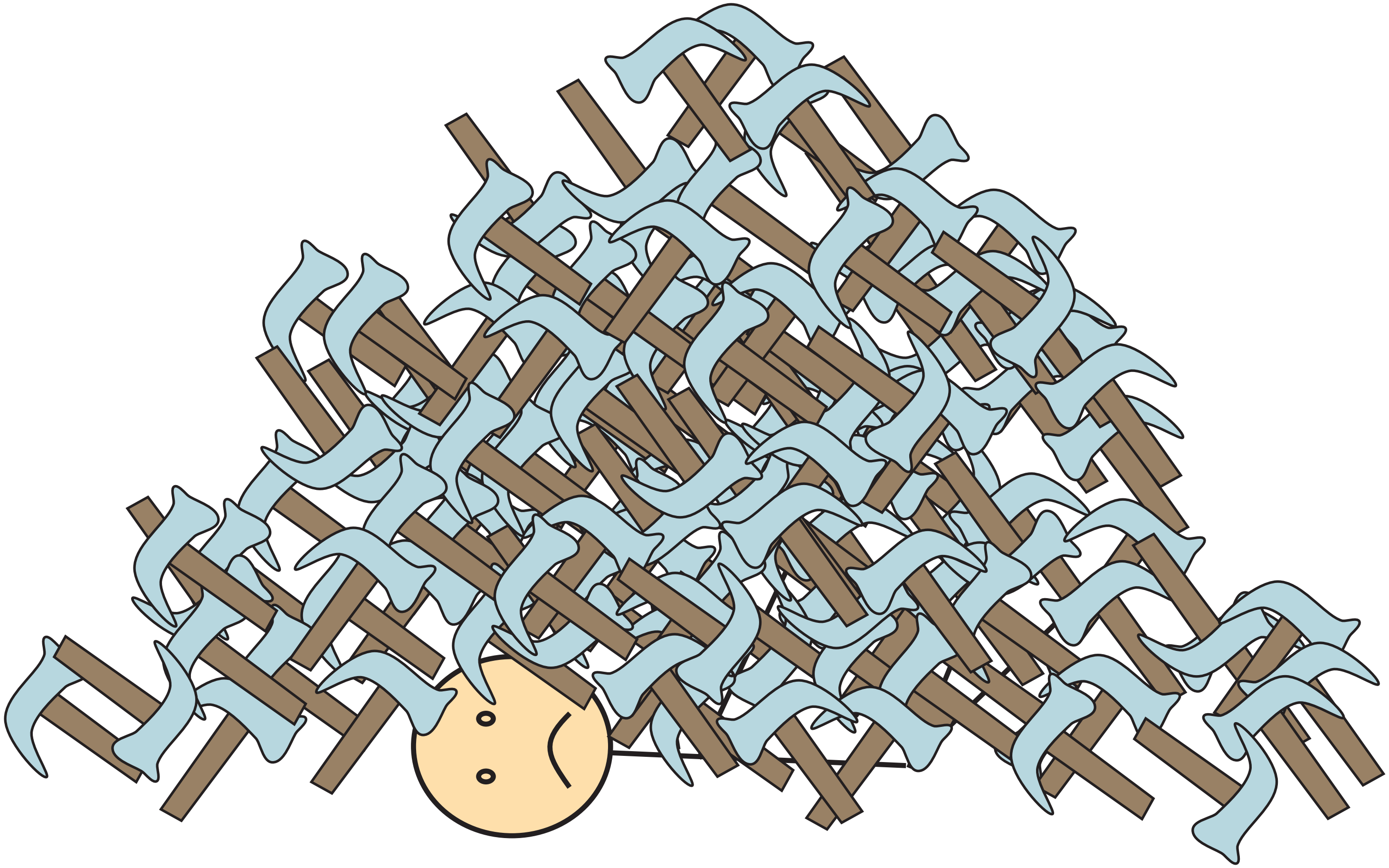

Hard work?

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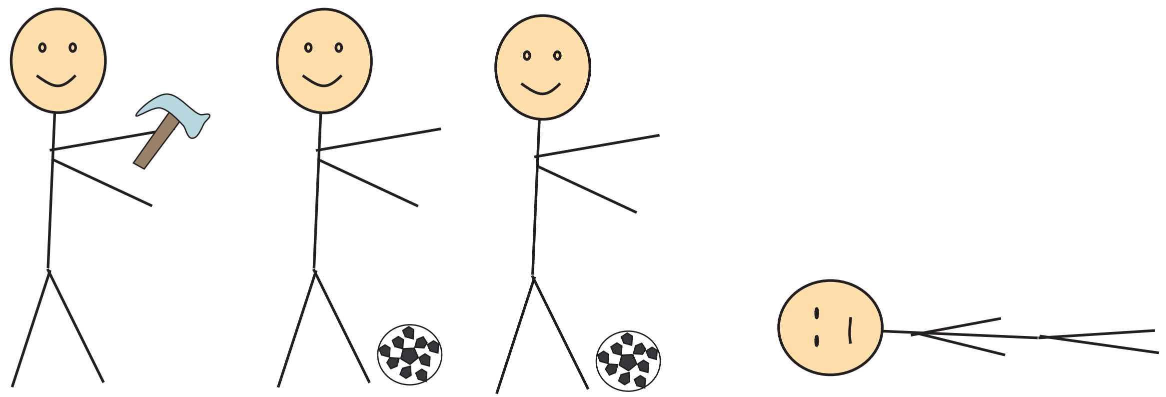
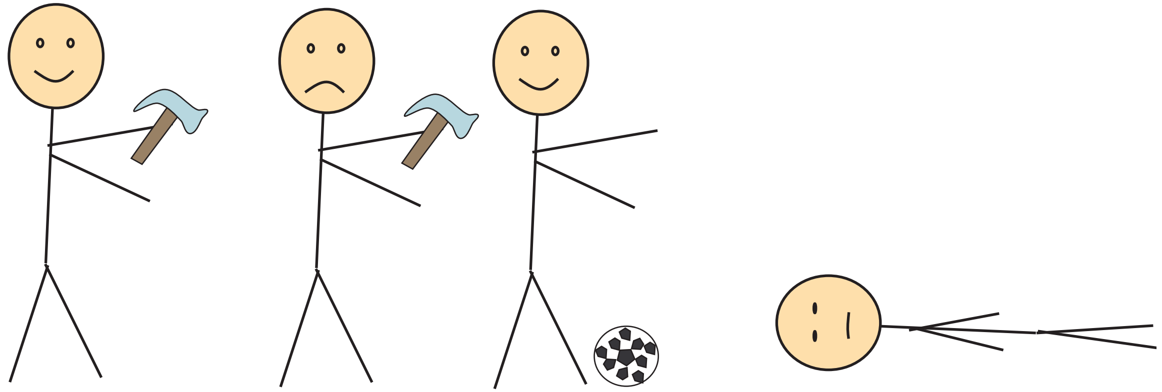


Hard work?



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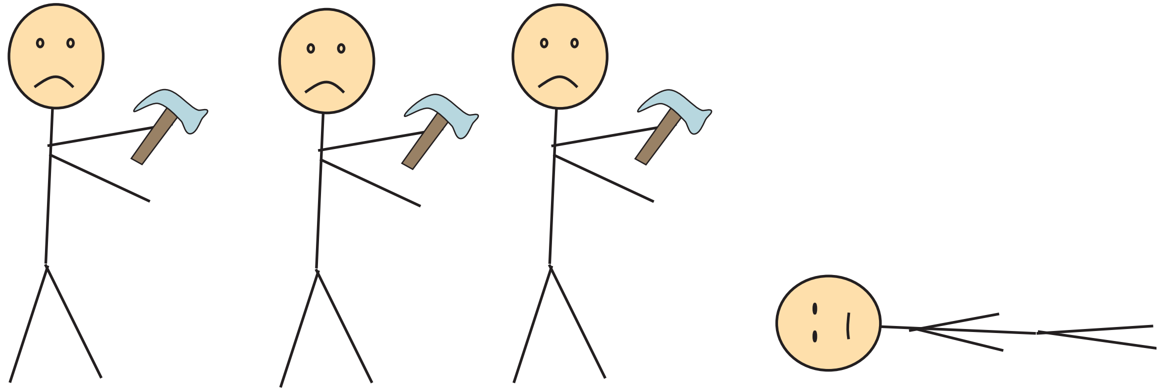

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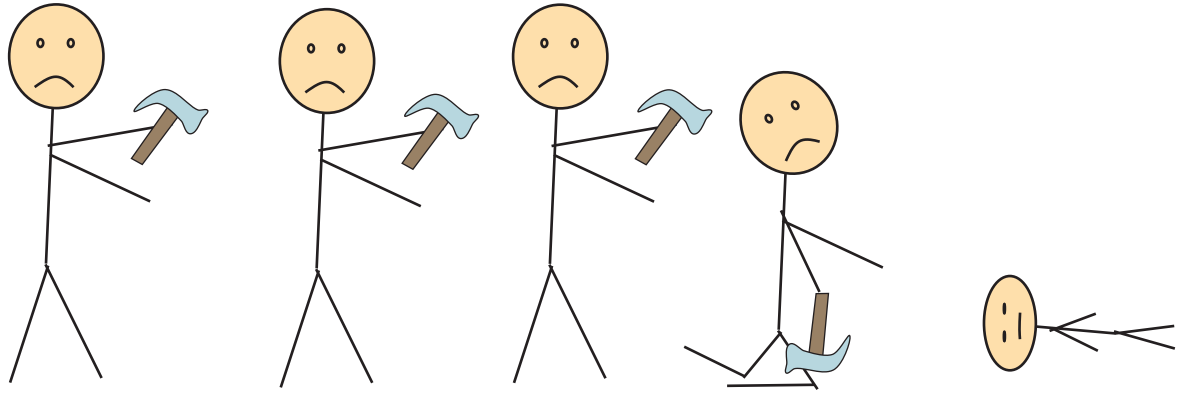
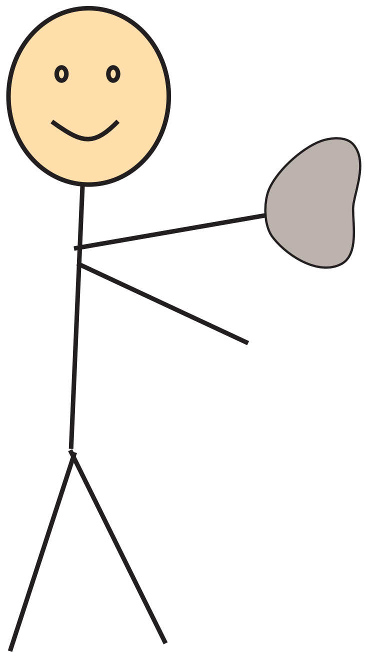
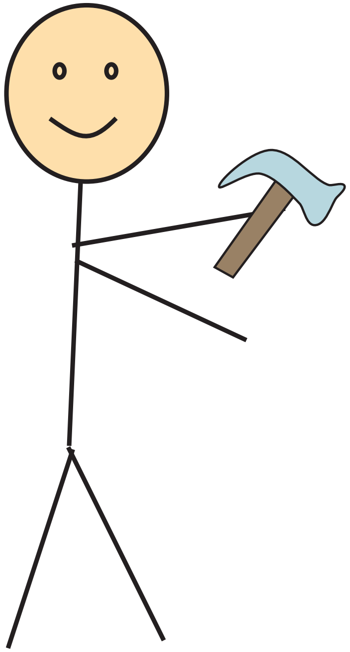


Technological progress?

Technological progress?



Technological progress?



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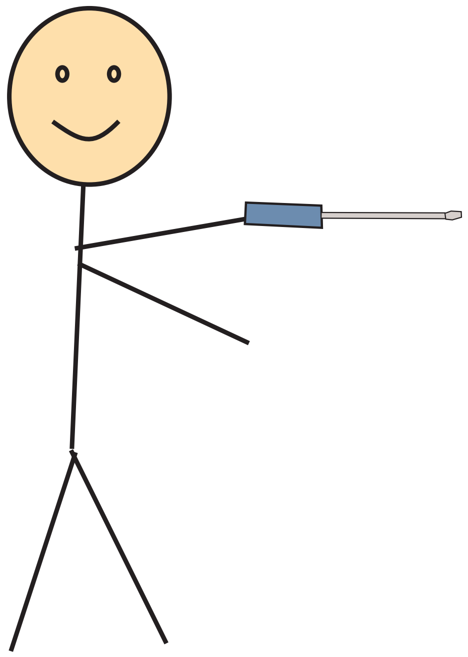

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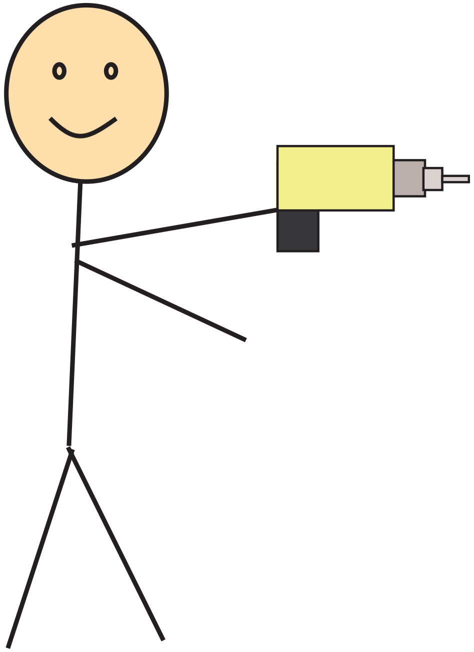

Technological progress?

Solow (1956) demonstrated, through mathematical reasoning similar to the above, that technological progress—which raises the productivity of inputs such as labour and capital—is the fundamental driver of growth.

Technological progress?

Solow (1956) demonstrated, through mathematical reasoning similar to the above, that technological progress—which raises the productivity of inputs such as labour and capital—is the fundamental driver of growth.

Solow's model is in continuous time, but our pictures suggest a simple approach in discrete time with *technology vintages* in which each newly discovered technology is implemented through investment in new capital, replacing the old.

Leading economies must both invent and invest, but those playing catch-up only need to invest.

Neoclassical growth models

Basics

Basics

Neoclassical growth models are workhorse models in economics. They are built around a *production function*, plus assumptions about how capital accumulates, how technology is developed and spreads through the economy, and finally how households behave.

The Solow model is a special case of a neoclassical growth model, as is the Ramsey model.

What is a production function?

What is a production function?

A production function is an equation relating inputs—which in the neoclassical model are labour and capital—to output of the final good.

The following equations are examples of general production functions:

$$Y = A_L L,$$
$$Y = F(A_L L, A_K K),$$
$$Y = F(A_L L, A_K K, A_R R).$$

If L is labour, K is capital, and R is a natural resource, what can we say about the units of the variables in these functions?

What is a production function?

If we are to add together two quantities, they must be in the same units. If we have 1 unit of steel and 1 unit of apples we cannot say that we have two units of stuff altogether if steel is measured in tons and apples in kilos. We would have 1001 kilos of stuff, or 1.001 tons.

Since the same follows for subtraction, it follows that the units of the quantities on either side of any equation must be the same, since if we have $X = Y$ we can always write $X - Y = 0$.

What is a production function?

Let's go back to the previous equations:

$$
\begin{aligned}
Y &= A_L L, \\
Y &= F(A_L L, A_K K), \\
Y &= F(A_L L, A_K K, A_R R).
\end{aligned}
$$

If Y has units of *widgets/year*, and L has units of *workers*, then A_L must have units of *widgets /worker /year*. Labour productivity!

And if K has units of *widgets* (the stock of foregone consumption) then A_K must have units of *widgets /widget /year*. It is the productivity of unconsumed widgets in producing new widgets. Furthermore, the function F must be *constant returns*.

What is a production function?

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Why must F be 'constant returns'? What does this mean?

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If R is the flow of a natural resource such as iron ore, suggest units for R and A_R.

Assumptions of the neoclassical model

Assumptions of the neoclassical model

The production function:

$$Y = F(A_K K, A_L L).$$

$$F'_K, F'_L \geq 0.$$

$$F''_K, F''_L \leq 0.$$

Constant returns.

Assumptions of the neoclassical model

Technology:

Nonrival. Nonexcludable. Exogenous.

Assumptions of the neoclassical model

Capital accumulation and consumption:

$$K_{t+1} = (1-\delta)K_t + I_t,$$
$$Y_t = C_t + I_t.$$

Assumptions of the neoclassical model

Households?

The Solow and Ramsey models

Solow (with constant technology)

Solow (with constant technology)

The Solow model is a version of the neoclassical growth model in which households use a simple rule of thumb to determine their saving rate. Furthermore, the production function is Cobb–Douglas.

$$K_{t+1} = (1-\delta)K_t + sY_t.$$
$$Y = (A_L L)^{1-\alpha} K^{\alpha}.$$

Cobb–Douglas properties? Elasticity of substitution?

The elasticity of substitution between inputs is the sensitivity of the relative quantities of the inputs (K/L) to their relative prices w_K/w_L.[1]

[1] We can also use L/K and w_L/w_K, it makes no difference.

Solow (with constant technology)

$$Y = (A_L L)^{1-\alpha} K^{\alpha}.$$

Set up the profit-maximization problem of a representative final-good producer, and take first-order conditions (FOCs) in the input prices. Eliminate Y from these two equations and find η where

$$\eta = -\frac{\partial(K/L)}{\partial(w_K/w_L)} \cdot \frac{w_K/w_L}{K/L}.$$

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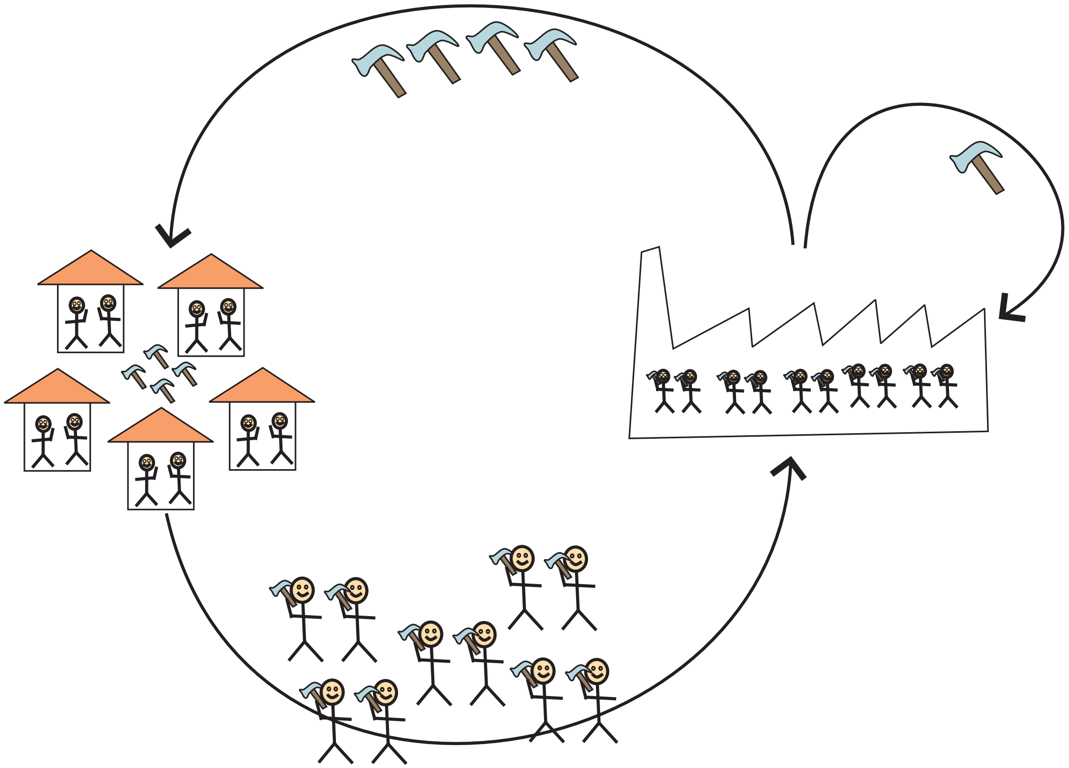
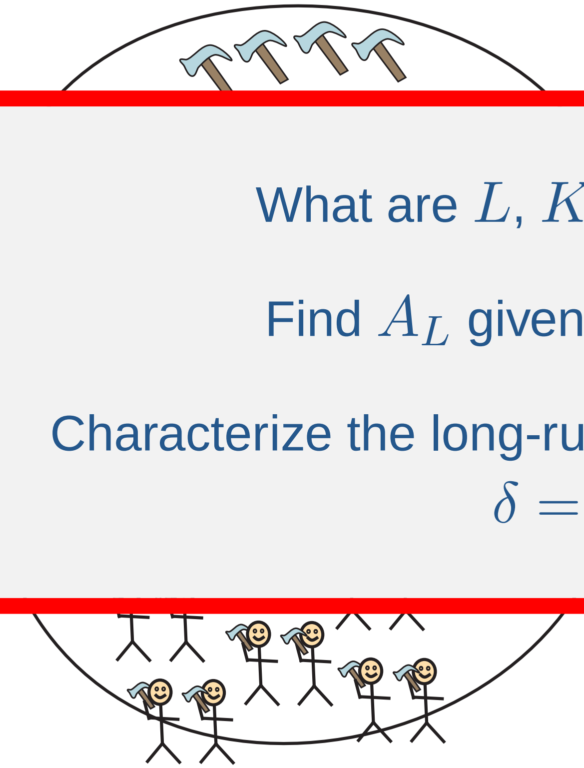
Assume that the wage rises by 1 percent relative to the price of capital. What happens to the factor share of labour? Discuss!

Solow (with constant technology)



The picture illustrates a Solovian economy in which the final good is hammers. Take it literally and answer the following.

Solow (with constant technology)



What are L, K, Y, C, and s?

Find A_L given that $\alpha = 1/3$.

Characterize the long-run steady state given that $\delta = 0.1$.

The picture illustrates a Solovian economy in which the final good is hammers. Take it literally and answer the following.

Solow (with constant technology)

Discuss effects of different saving rates across countries.

And if there is a global capital market?

Solow with exogenous growth in A_L

Solow with exogenous growth in A_L

Before working through this section, you might want to work through the mathematical appendix.

Solow with exogenous growth in A_L

Solow with exogenous growth in A_L

$$
\begin{aligned}
Y &= (A_L L)^{1-\alpha} K^{\alpha} \\
\dot{A}_L / A_L &= g_{A_L} \\
\dot{L}/L &= n \\
\dot{K} &= sY - \delta K.
\end{aligned}
$$

On b.g.p., $\dot{K}/K$ constant, hence $sY/K - \delta$ constant, hence Y/K constant, hence $(A_L L/K)^{1-\alpha}$ constant, and

$$\dot{K}/K = \dot{A}_L/A_L + \dot{L}/L.$$

$y = Y/L$. What is $\dot{y}/y$?

Solow with exogenous growth in A_L

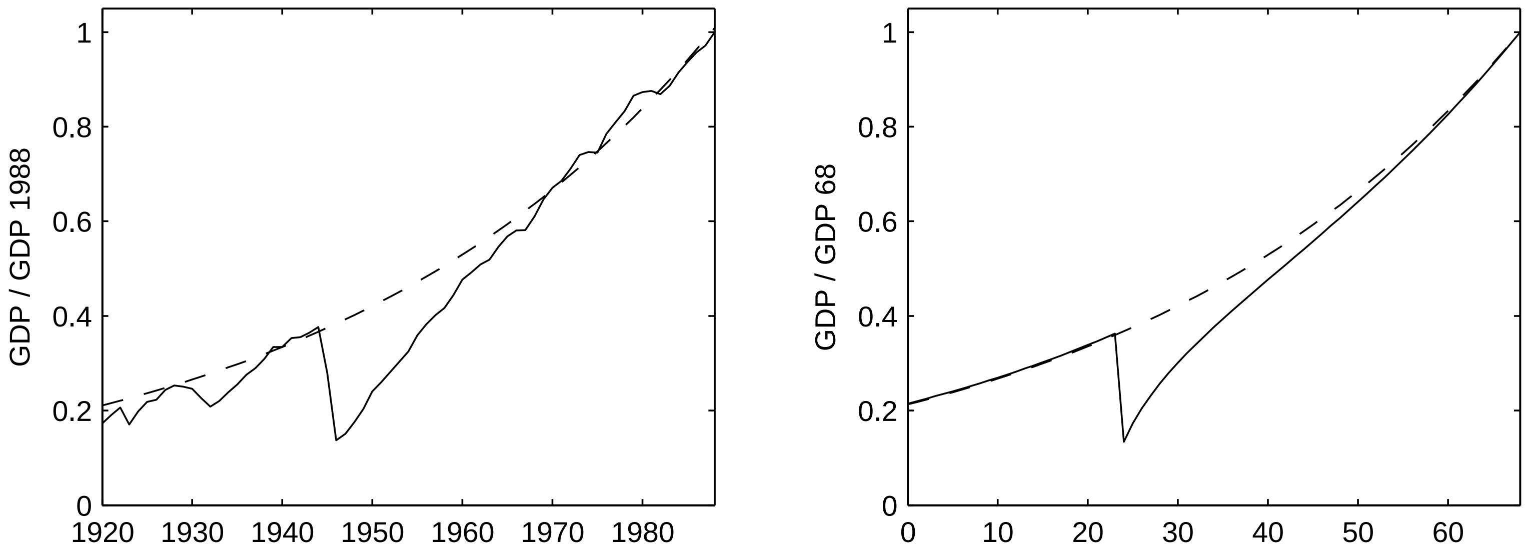


Figure 1: Testing the Solow model. Upper panel: German real GDP, compared to an estimated constant-growth trend (growth rate 2.3 percent per year). Lower panel: Model simulation assuming the same growth trend in labour productivity and parameters $\alpha = 0.3$, $\delta = 0.1$, $s = 0.2$, with a massive shock to capital (the stock is divided by a factor of 30 at the end of year 24).

Solow with exogenous growth in A_L

What is the Solow model good for?

What drives growth in the Solow model?

What drives growth in real economies?

Ramsey

Ramsey

In Ramsey (or Ramsey–Cass–Koopmans) we endogenize the savings rate based on a simple model of household preferences.

$$\max \sum_h U^h,$$

where $$U^h = \sum_t u^h(c_t)\beta^t.$$

and $$u = \frac{c^{1-\sigma} - 1}{1-\sigma}.$$

What is the Ramsey model good for?

For more, including links, see Wiki.

No capital

No capital

$$Y = A_L L[K/(A_L L)]^\alpha,$$

and note that in balanced growth K grows at the same rate as $A_L L$, so $K/(A_L L)$ is constant, then it should be clear that in long-run analysis we lose little by simplifying the production function to

$$Y = A_L L.$$

The same conclusion applies when we include resources in the production function. Then we have (for instance)

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\beta R^\alpha \\ &= (A_L L)^{1-\alpha}[K/(A_L L)]^\beta R^\alpha. \end{aligned}$$

What do we lose by dropping capital? Gain?

Endogenous growth

Key literature

How to model deliberate investment in research and development of new technology, which raises the productivity of inputs?

If this technology is then available to all—free to copy—then the investor cannot make money from it. So the investment won't be made in the first place. We need some form of *market power* over the technology.

The first to develop a model including such a mechanism was Romer (1990), Journal of Political Economy. Aghion and Howitt (1992), Econometrica, followed up with a model of *creative destruction* in which new vintages replaced the old.

In the book we develop a multi-sector model that has similarities to the Aghion and Howitt approach. Here I take you through some key intuition.

A multi-sector model

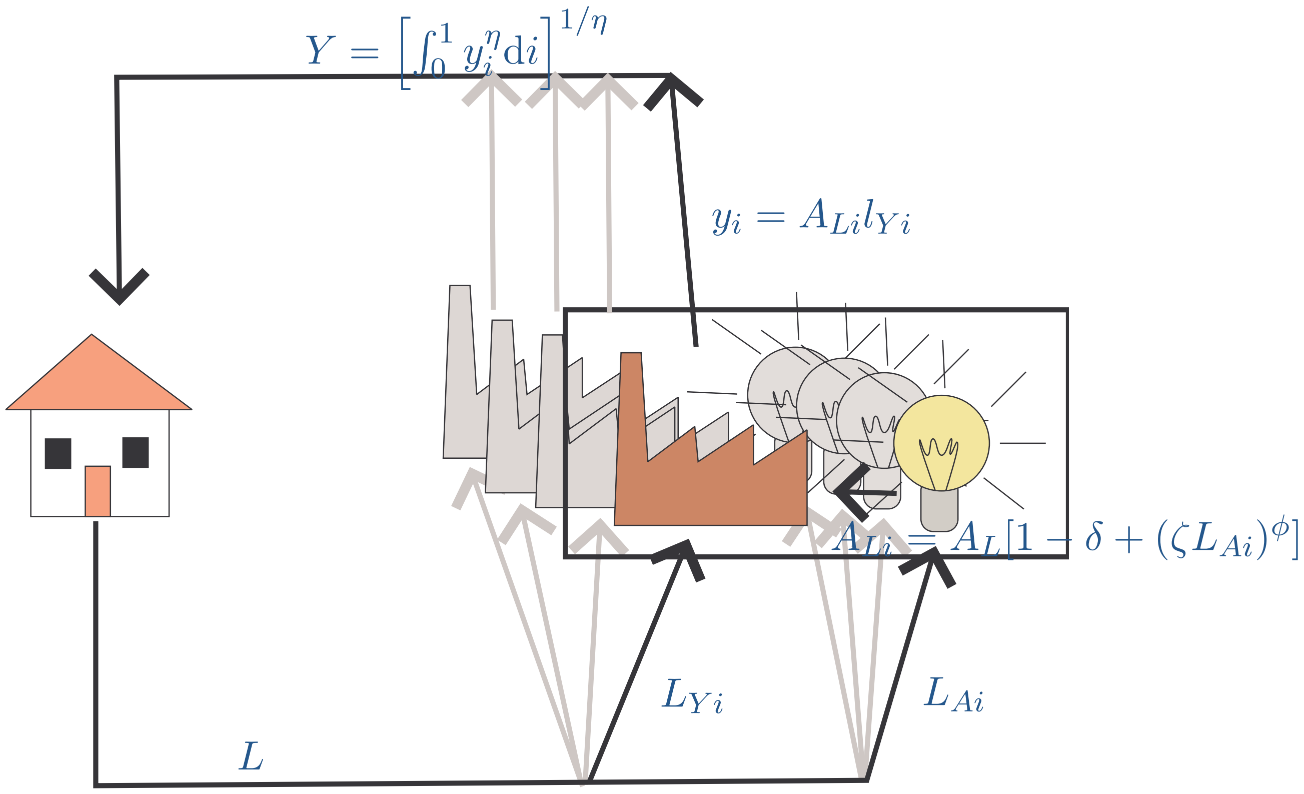
Assume you own a firm with product y_j competing with a unit mass of other firms with different products, selling to competitive final-good producers who put all the products together to make a final good Y (price normalized to 1). The final-good producer's production function is CES:

$$Y = \left[\int_0^1 y_i^\eta \, \mathrm{d}i\right]^{1/\eta},$$

where $\eta \in (0, 1)$. To make your product you hire production labour L_{Yj} on a competitive labour market, but you can also hire (the same) labour to do research L_{Aj}, boosting productivity A_{Lj}:

$$y_j = A_{Lj} L_{Yj};$$
$$A_{Lj} = A_L[1 - \delta + (\zeta L_{Aj})^\phi].$$

A multi-sector model



A multi-sector model

For each individual firm there is an optimal balance between production labour and research labour, to produce its product as cheaply as possible.

The more each firm invests in research, the more knowledge is created, which spills over to the other firms in future periods (through A_L, general knowledge), driving growth.

Because of the limited labour force, firm size is determined and the wage arises through the competition of firms for workers.

The parameter η is at a level such that optimizing firms break even when there is a unit mass of firms. If η were lower (closer to zero) firms would have more market power, hence they would make positive profits and more firms would enter. If η were higher (closer to 1) the reverse would be true, firms would exit.

Endogenizing growth

Endogenizing growth

We can take an ‘exogenous growth model’ in which workers devote a fixed proportion of their time to research for no obvious reason, and turn it into an endogenous growth model in which research time is the result of competing firms solving optimization problems.

In the endogenous growth model, research is *undersupplied* compared to the social optimum, because (i) researchers can’t capture all of the current social benefits through their (limited) market power, and (ii) their knowledge builds the foundation for future researchers who ‘stand on the shoulders of giants’ (as Newton said).

So research should be subsidized! E.g. Jones and Williams (2000).

We will use the endogenous model in later chapters when we wish to analyse how firms choose between investment in alternative types of technology, e.g. ‘clean’ and ‘dirty’.

Mathematical appendix

Differentiation and the chain rule

Differentiation and the chain rule

How can we differentiate $(x^2)^3$. We could of course say that $y = (x^2)^3 = x^6$, hence $\mathrm{d}y/\mathrm{d}x = 6x^5$. However, we could also use the chain rule. First differentiate 'the outside' treating the inside as constant, then multiply by the differential of 'the inside':

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\left(x^2\right)^2 \times 2x = 6x^5.$$

Now consider $y = (2x+1)^3$. We could expand the RHS and then differentiate. Or we could use the chain rule:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3(2x+1)^2 \times 2 = 6(2x+1)^2.$$

Soon we will look at cases where we *must* use the chain rule. But first, how to differentiate *like an economist*.

Differentiation like an economist

Differentiation like an economist

Image we have the following expression, and we want to find $\mathrm{d}y/\mathrm{d}x$:

$$y = \log A \sin B(1 + \cos^2 C)/(1 - D^2)e^Q x.$$

We could write $\mathrm{d}y/\mathrm{d}x = \log A \sin B(1 + cos^2 C)/(1 - D^2)e^Q$, or we could 'differentiate like economists' and write $\mathrm{d}y/\mathrm{d}x = y/x$. The latter is of course much simpler, especially so if we are ultimately interested in an elasticity (which we often are as economists) so

$$\eta = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{x}{y} = 1.$$

Combining the chain rule and DLAE

Combining the chain rule and DLAE

Now consider

$$y = [\gamma x_1^\epsilon + (1-\gamma)x_2^\epsilon]^{1/\epsilon}.$$

How do we differentiate this with respect to x_1 (or x_2)? Clearly we need the chain rule, but we should also do it 'like economists'. With the chain rule alone we have

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{\epsilon}[\gamma x_1^\epsilon + (1-\gamma)x_2^\epsilon]^{1/\epsilon-1}\gamma\epsilon x_1^{\epsilon-1} \\ &= [\gamma x_1^\epsilon + (1-\gamma)x_2^\epsilon]^{(1-\epsilon)/\epsilon}\gamma x_1^{\epsilon-1} \\ &= \gamma x_1^\epsilon y^{1-\epsilon}/x_1.\end{aligned}$$

But if we 'differentiate like economists' we can go straight to the final result!

Balanced growth

Balanced growth

In an economy with economic growth we call growth *balanced* if all the state variables (things like capital, labour, productivity indices, etc.) are growing at *constant rates*. These rates may differ from each other, and may be positive, negative, or zero.

Often our model economies will move towards a *balanced growth path* (bgp) over time, hence that path can teach us a lot about what happens in the economy in the (very) long run.

Analysing the bgp is almost always much simpler than analysing the *transition path*, which is how we move towards the bgp from any given starting point.

Since we are mainly interested in the long run in this course, we often focus on balanced growth paths and not transition paths.

Balanced growth

What is a growth rate, mathematically? A variable Y is growing at a constant (or *exponential*) rate g if we can write

$$Y(t) = e^{gt}.$$

Now differentiate (like an economist) wrt time t to obtain

$$\frac{\partial Y}{\partial t} = \dot{Y} = gY.$$

Hence

$$\dot{Y}/Y = g.$$

So the growth rate of a variable Y is defined as $\dot{Y}/Y$.

Balanced growth

To characterize a bgp we often start by assuming that it exists. Imagine for instance an economy where one of the equations is

$$\dot{Y} = X_1 - kY,$$

where k is constant. Now, if we are on a bgp then we know that $\dot{Y}/Y$ is constant, hence

$$X_1/Y - k$$

must also be constant. The only way this can hold is that $\dot{X}_1/X_1 = \dot{Y}/Y$. Both the variables grow at equal (constant) rates!

Differentiation with respect to time

Differentiation with respect to time

Now assume an economy (which may or may not be on a bgp) in which $Y(t) = X(t)$, or[1] just $Y = X$. What can we say about $\dot{Y}$? Clearly, $\dot{Y} = \dot{X}$: if Y and X are always equal, they must grow at equal rates. But what about

$$Y = X^2?$$

Then we need to use the chain rule again, in a slightly different way:

$$\dot{Y} = \frac{\partial Y}{\partial t} = \frac{\partial Y}{\partial X}\frac{\partial X}{\partial t} = 2X\dot{X} = 2\frac{Y}{X}\dot{X}.$$

Hence $\dfrac{\dot{Y}}{Y} = 2\dfrac{\dot{X}}{X}$. Note: DLAE!

[1] Note that we will often leave out the (t); you are expected to understand which variables are functions of time without it needing to be pointed out in each equation.

Differentiation with respect to time

What about

$$Y = AX_1^{1-\alpha-\beta}X_2^{\alpha}X_3^{\beta}?$$

What is $\dot{Y}/Y$, assuming that A, X_1, X_2, and X_3 are all functions of time? Then we know that

$$\frac{\partial Y}{\partial t} = \frac{\partial Y}{\partial A}\frac{\partial A}{\partial t} + \frac{\partial Y}{\partial X_1}\frac{\partial X_1}{\partial t} + \dots$$

Differentiating like an economist we find

$$\dot{Y} = \frac{Y}{A}\dot{A} + (1-\alpha-\beta)\frac{Y}{X_1}\dot{X}_1 \dots$$

hence $$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + (1-\alpha-\beta)\frac{\dot{X}_1}{X_1} + \alpha\frac{\dot{X}_2}{X_2} + \beta\frac{\dot{X}_3}{X_3}.$$