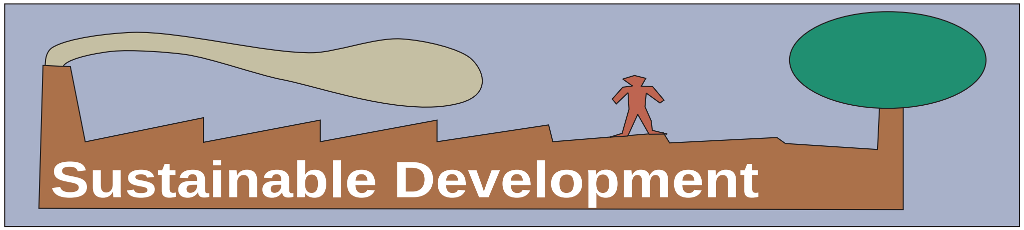
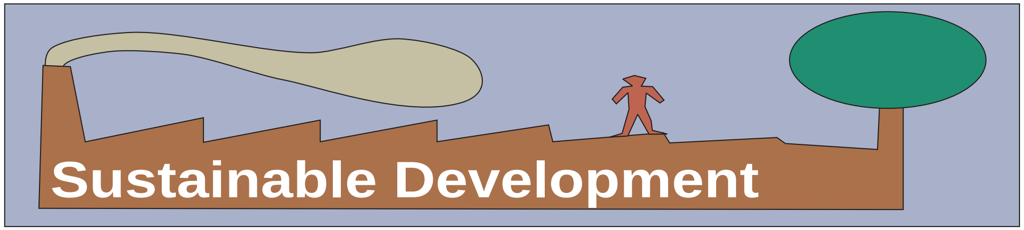




Chapter 5

DTC and resource efficiency

Resource efficiency in the production function

Resource efficiency in the production function

$$y = F(A_L L, A_R R).$$

$$y = (A_L L)^{\alpha}(A_R R)^{1-\alpha} = AL^{\alpha}R^{1-\alpha}.$$

$$y = \min\{A_L L, A_R R\}.$$

$$y = [\gamma(A_L L)^{\epsilon} + (1-\gamma)(A_R R)^{\epsilon}]^{1/\epsilon}.$$

Resource efficiency in the production function

$$y = (A_L L)^\alpha (A_R R)^{1-\alpha} = A L^\alpha R^{1-\alpha}.$$

Why not Cobb–Douglas?

Resource efficiency in the production function

$$y = \min\{A_L L, A_R R\}.$$

Advantage of Leontief. Think of making hammers from steel.

Resource efficiency in the production function

$$y = [\gamma(A_L L)^\epsilon + (1-\gamma)(A_R R)^\epsilon]^{1/\epsilon}.$$

What values are possible for ϵ?

Discuss special cases.

Advantages of CES?

Resource efficiency in the production function

Assume an economy with 10 people on an island, and 10 trees/week wash up on the beach. Furthermore, the islanders have a technology called ‘knives’ which allows them to cut the trees into planks, which can rapidly be made into houses (final product). They manage to make 0.01 houses per week.

What do they need more of to boost their rate of housebuilding? Suggest values for the elasticity of substitution between the inputs, and knowledge levels. Explain briefly.

Resource efficiency in the production function

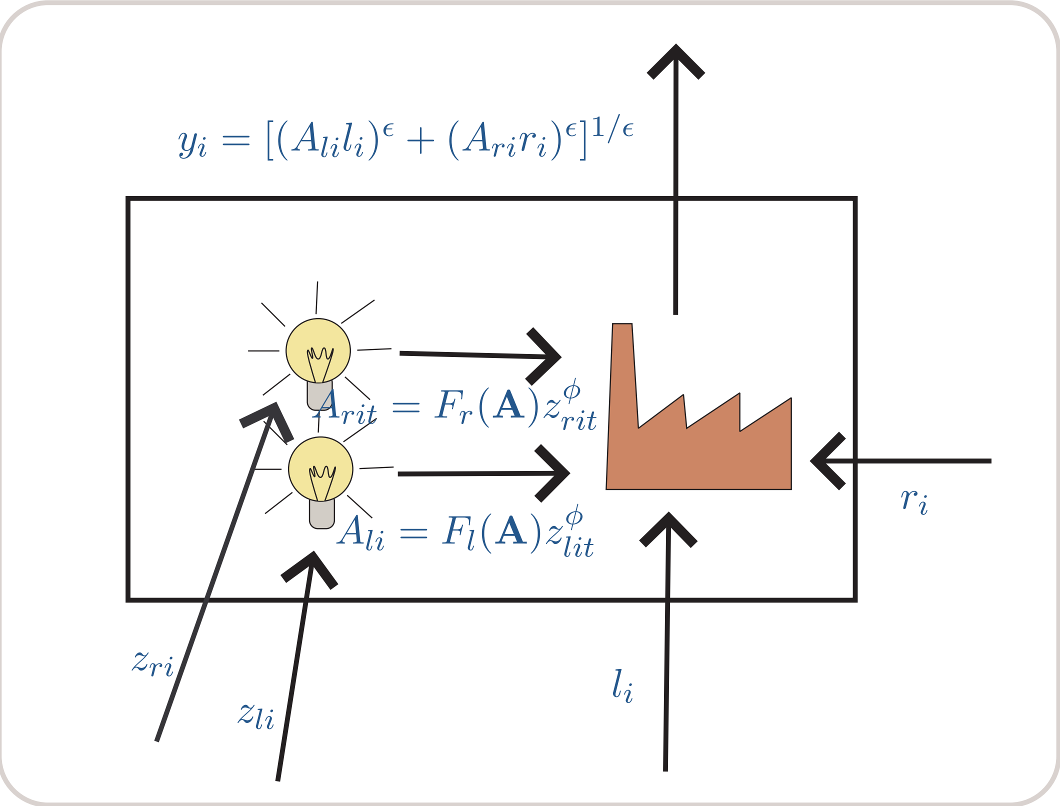
Now assume that the islanders invent a technology called ‘sawmills’ (and are somehow able to obtain the necessary capital goods). What do they need more of now in order to boost their rate of housebuilding? Suggest new values for the knowledge levels. Explain briefly.

Foundations of an aggregate DTC model

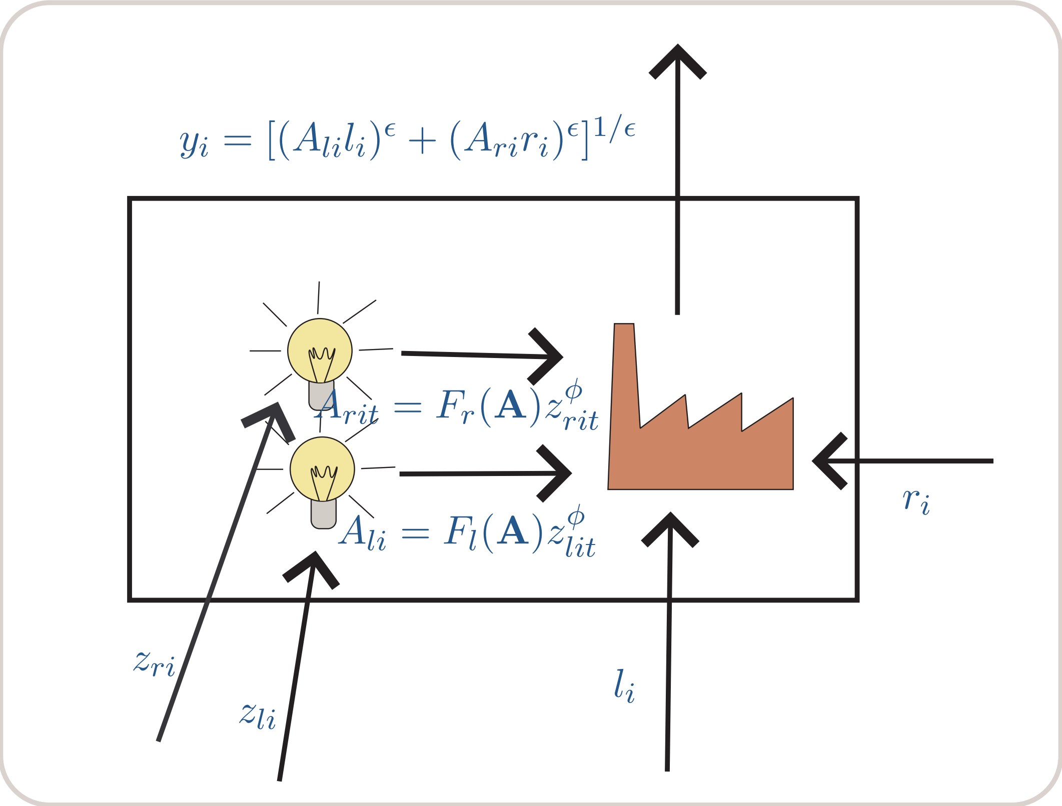
Foundations of an aggregate DTC model

Focus an a single firm's production function.

Foundations of an aggregate DTC model



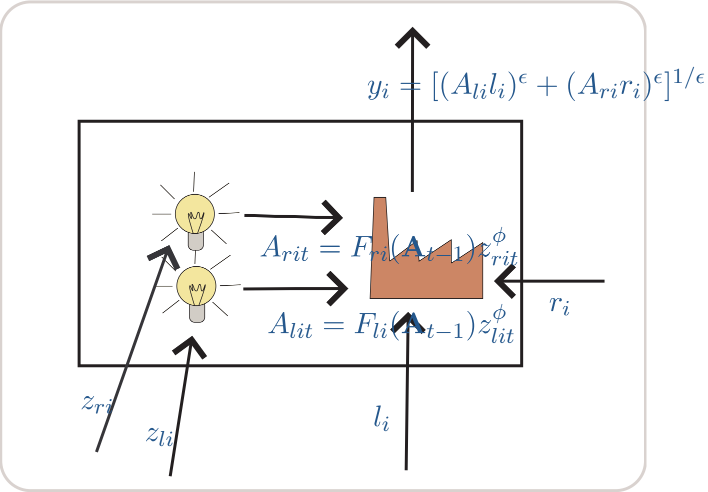
Foundations of an aggregate DTC model



Assume

$$\frac{\partial p_i}{\partial y_i}\frac{y_i}{p_i} = -(1-\eta).$$

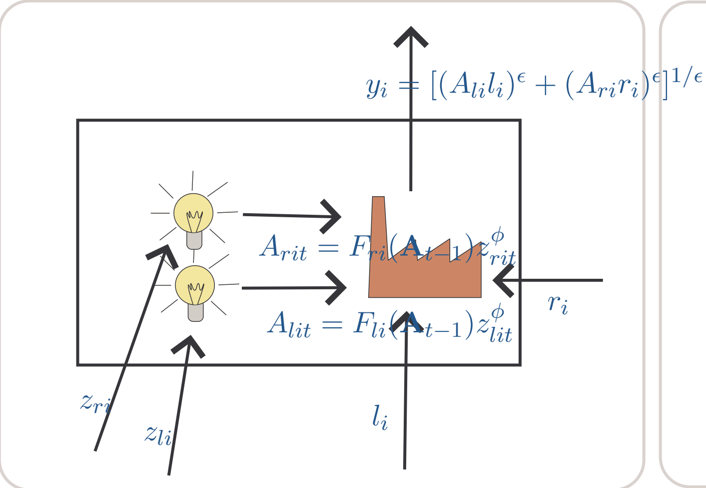
Foundations of an aggregate DTC model



Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i) - \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model

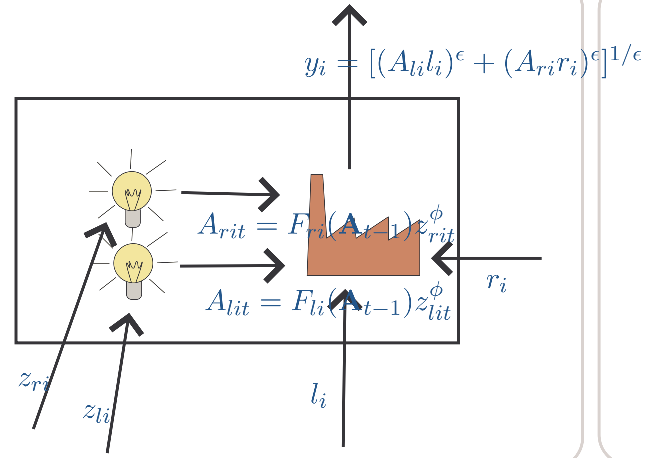


Take FOCs in l_i and r_i

Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i) - \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model

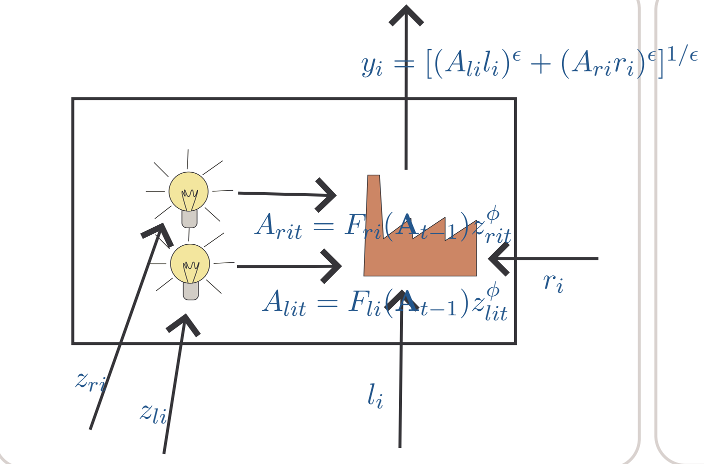


$$\frac{w_L l_i}{w_R r_i} = \left(\frac{A_{li} l_i}{A_{ri} r_i}\right)^\epsilon$$

Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i)$$
$$- \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model



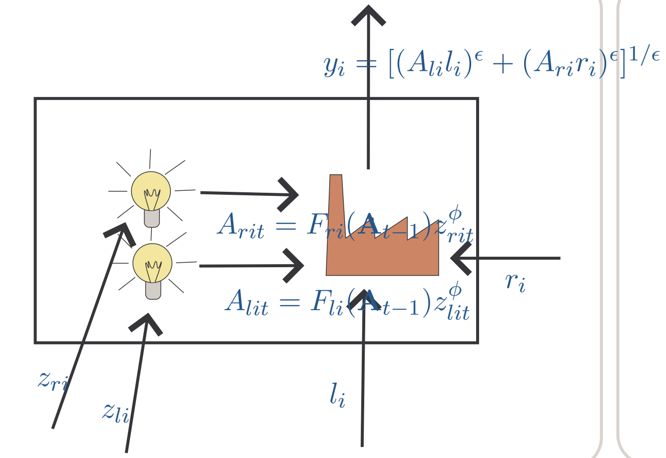
$$\frac{w_L l_i}{w_R r_i} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon$$

Take FOCs in A_{li} and A_{ri}

Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i)$$
$$- \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model



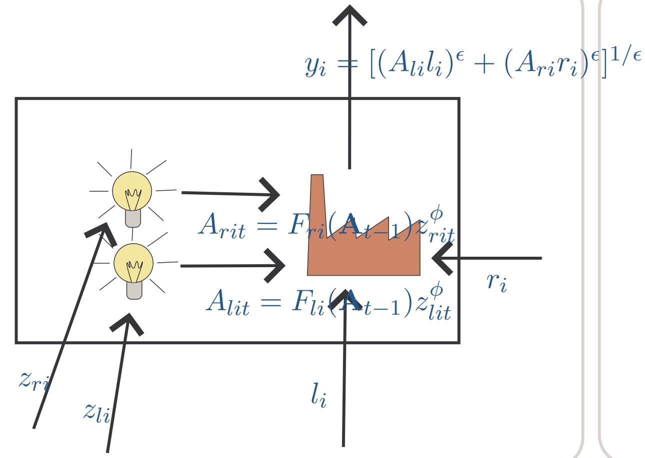
$$\frac{w_L l_i}{w_R r_i} = \left(\frac{A_{li} l_i}{A_{ri} r_i}\right)^\epsilon$$

$$\frac{\lambda_{li} A_{li}}{\lambda_{ri} A_{ri}} = \left(\frac{A_{li} l_i}{A_{ri} r_i}\right)^\epsilon$$

Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i) - \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model



$$\frac{w_L l_i}{w_R r_i} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon$$

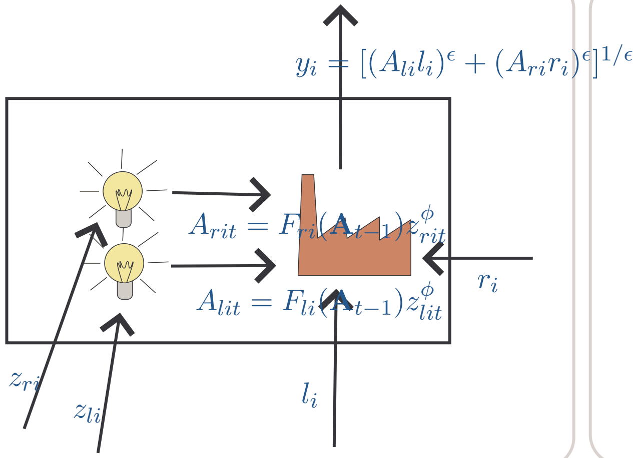
$$\frac{\lambda_{li}A_{li}}{\lambda_{ri}A_{ri}} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon$$

Take FOCs in z_{li} and z_{ri}

Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i) \\ - \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

Foundations of an aggregate DTC model



$$\frac{w_L l_i}{w_R r_i} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon$$

$$\frac{\lambda_{li}A_{li}}{\lambda_{ri}A_{ri}} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon$$

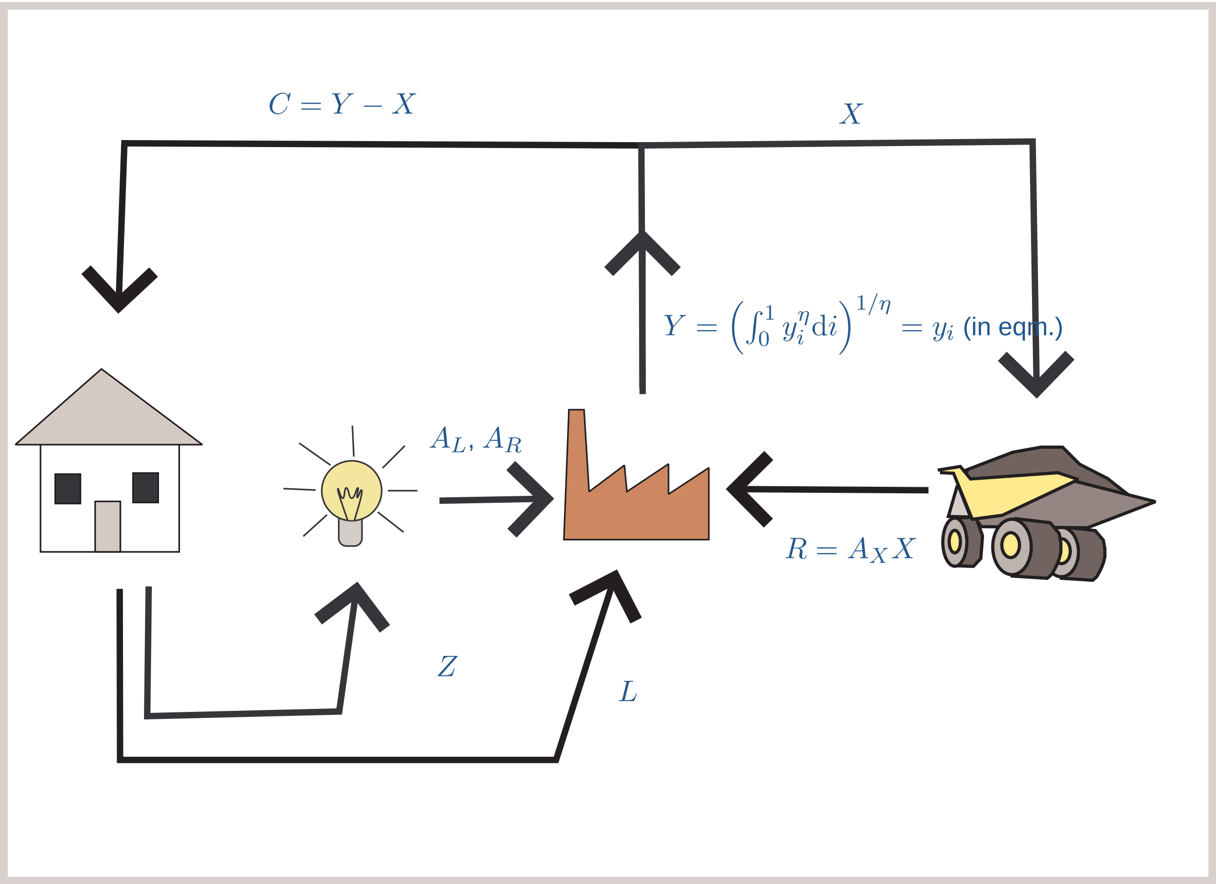
$$\frac{z_{li}}{z_{ri}} = \frac{\lambda_{li}A_{li}}{\lambda_{ri}A_{ri}}$$

$$\frac{z_{li}}{z_{ri}} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^\epsilon = \frac{w_L l_i}{w_R r_i}$$

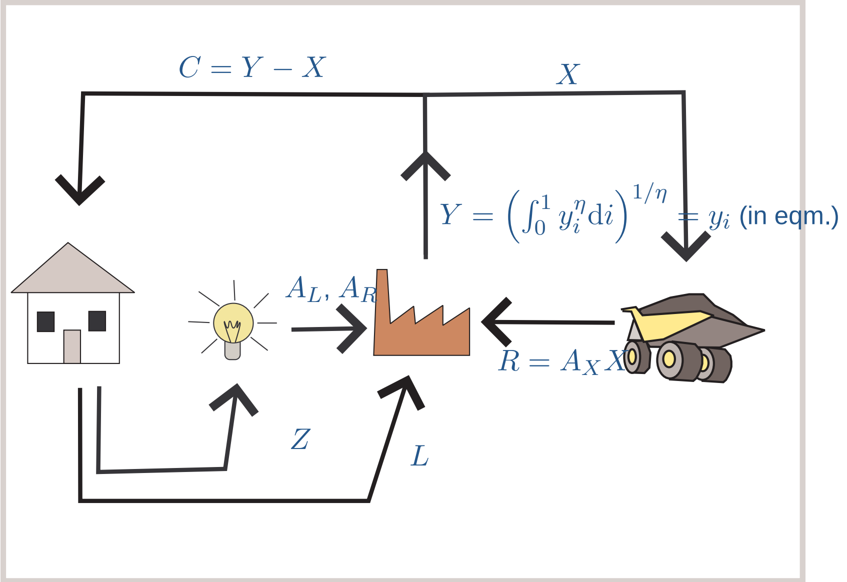
Lagrange:

$$\mathcal{L} = p_i(y_i)[(A_{li}l_i)^\epsilon + (A_{ri}r_i)^\epsilon]^{1/\epsilon} - w_z(z_{li} + z_{ri}) - (w_l l_i + w_r r_i) - \lambda_{li}(A_{li} - F_l \cdot z_{li}^\phi) - \lambda_{ri}(A_{ri} - F_r \cdot z_{ri}^\phi).$$

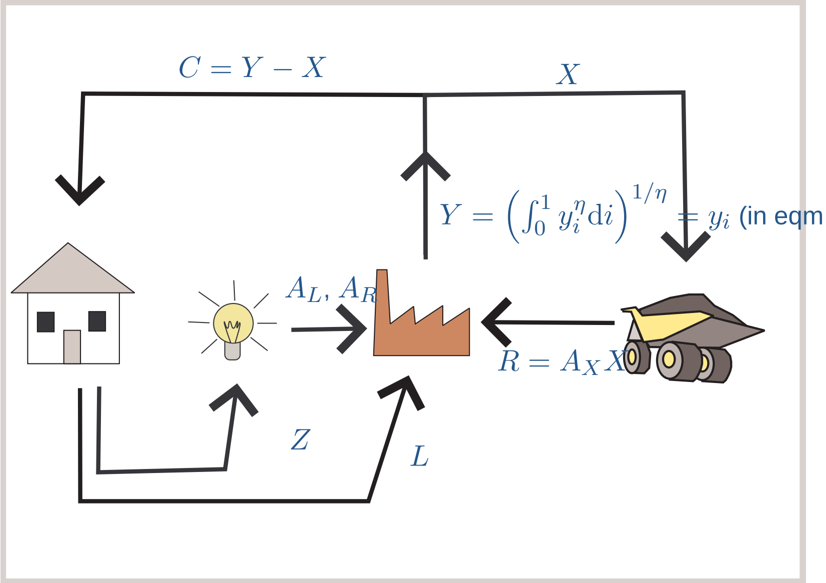
The aggregate model



The aggregate model



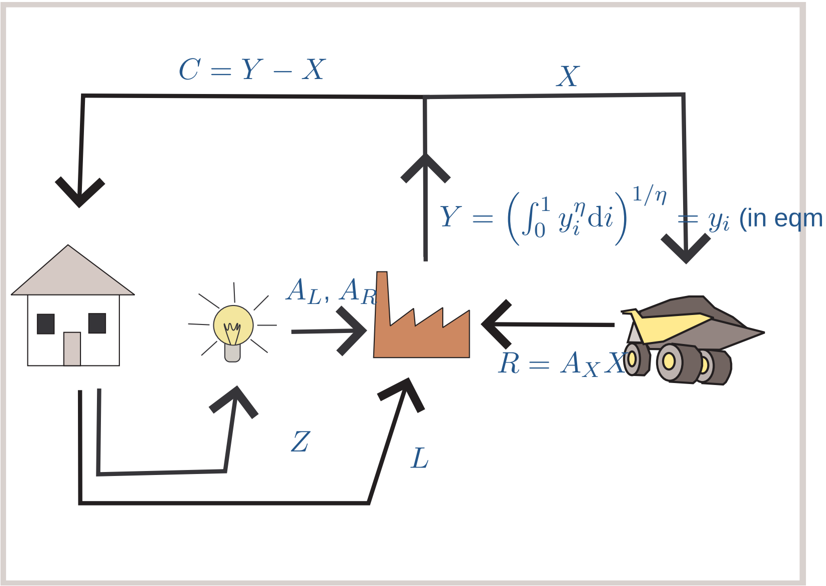
The aggregate model



$$\frac{z_{li}}{z_{ri}} = \left(\frac{A_{li} l_i}{A_{ri} r_i}\right)^\epsilon = \frac{w_L l_i}{w_R r_i}$$

$$\frac{Z_L}{Z_R} = \left(\frac{A_L L}{A_R R}\right)^\epsilon = \left(\frac{A_L / w_L}{A_R / w_R}\right)^{\epsilon/(1-\epsilon)} = \frac{w_L L}{w_R R}$$

The aggregate model



$$\frac{z_{li}}{z_{ri}} = \left(\frac{A_{li} l_i}{A_{ri} r_i}\right)^\epsilon = \frac{w_L l_i}{w_R r_i}$$

$$\frac{Z_L}{Z_R} = \left(\frac{A_L L}{A_R R}\right)^\epsilon = \left(\frac{A_L / w_L}{A_R / w_R}\right)^{\epsilon/(1-\epsilon)} = \frac{w_L L}{w_R R}$$

Assume $$A_{Lt} = (A_{Lt-1}/\zeta_L) Z_{Lt}^\phi$$

and $$A_{Rt} = (A_{Rt-1}/\zeta_R) Z_{Rt}^\phi,$$

so $$\frac{A_{Lt}}{A_{Rt}} = \frac{A_{Lt-1}}{A_{Rt-1}} \frac{\zeta_R}{\zeta_L} \left(\frac{Z_{Lt}}{Z_{Rt}}\right)^\phi.$$

The aggregate model

Assume economy on b.g.p. with constant Z_L/Z_R, hence (below) A_L/A_R changing at constant rate.

Assume w_L/w_R grows (exogenously) at a constant rate. Then (right) L/R must fall at the same rate. And (right) A_L/A_R must rise at that rate.

And we know that there is some level of Z_L/Z_R (and hence A_L/A_R) which yields this rate.

So a b.g.p. exists on which factor shares are constant!

$$\frac{z_{li}}{z_{ri}} = \left(\frac{A_{li}l_i}{A_{ri}r_i}\right)^{\epsilon} = \frac{w_L l_i}{w_R r_i}$$

$$\frac{Z_L}{Z_R} = \left(\frac{A_L L}{A_R R}\right)^{\epsilon} = \left(\frac{A_L/w_L}{A_R/w_R}\right)^{\epsilon/(1-\epsilon)} = \frac{w_L L}{w_R R}$$

Assume $$A_{Lt} = (A_{Lt-1}/\zeta_L)Z_{Lt}^{\phi}$$

and $$A_{Rt} = (A_{Rt-1}/\zeta_R)Z_{Rt}^{\phi},$$

so $$\frac{A_{Lt}}{A_{Rt}} = \frac{A_{Lt-1}}{A_{Rt-1}}\frac{\zeta_R}{\zeta_L}\left(\frac{Z_{Lt}}{Z_{Rt}}\right)^{\phi}.$$

The aggregate model

Is the b.g.p. stable?

Intuition. Assume the economy is on a b.g.p., and then there is a small shock such that the quantity of R increases more than expected (or the price of R decreases if prices are exogenous and quantities endogenous).

What happens to the factor share of R? Investment Z_R/Z_L? And $A_R R/(A_L L)$ in the long run?

The b.g.p. is stable, as long as $\epsilon < 0$.

The aggregate model

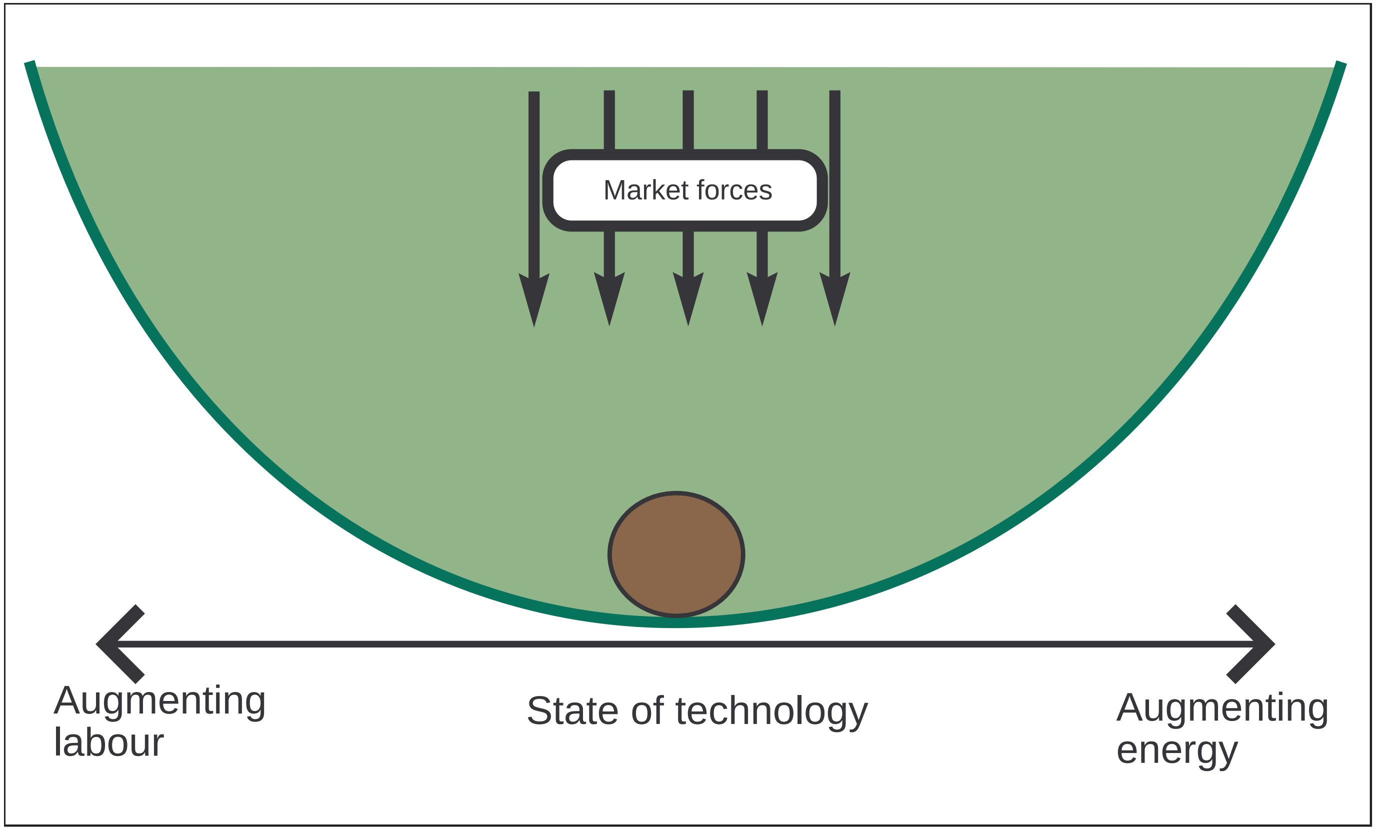


Figure 1: Illustration of how relative prices (the shape of the economic landscape) determine the relative levels of technology augmenting labour and energy.

The aggregate model

On a b.g.p.,

$$\frac{A_L L}{A_R R}$$

is constant. Therefore the factor shares are constant. The long-run aggregate production function 'appears to be' Cobb–Douglas!

$$y = AL^{1-\alpha} R^{\alpha}.$$

Problems regarding the DTC model

Problems regarding the DTC model

The model predicts that if w_R is constant while w_L increases, then A_R/A_L should decline.

We do not observe such a decline. In fact A_R seems to grow at least as fast as A_L, at least in the case of energy where there is clear evidence.

In the model, A_R is the efficiency of resources in making widgets. If it grows, R/Y (resources per widget) must decline. But R/Y has not declined historically. Why not?

Problems regarding the DTC model

The model of knowledge growth makes no sense.

$$A_{lit} = (A_{Lt-1}/\zeta_L) z_{lit}^{\phi};$$
$$A_{rit} = (A_{Rt-1}/\zeta_R) z_{rit}^{\phi}.$$

Growth in A_R should be linked (strongly) to the stock of labour-augmenting knowledge A_L.

If we put such links in the model then A_R grows even when w_R is constant, but the model no longer matches the quantity data since R/Y declines.