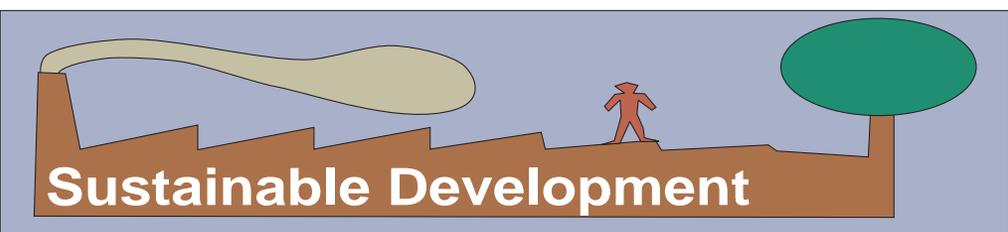
An illustration of a landscape with a brown ground, a green tree on the right, and a stylized orange figure in the center. A large, light-colored, curved shape resembling a cloud or smoke is on the left. The text "Sustainable Development" is written in white on a brown background at the bottom.

**Sustainable Development**

An identical illustration to the one on the left, featuring a landscape with a tree, a figure, and a cloud-like shape, with the text "Sustainable Development" at the bottom.

**Sustainable Development**

## Chapter 4

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# The DHSS model

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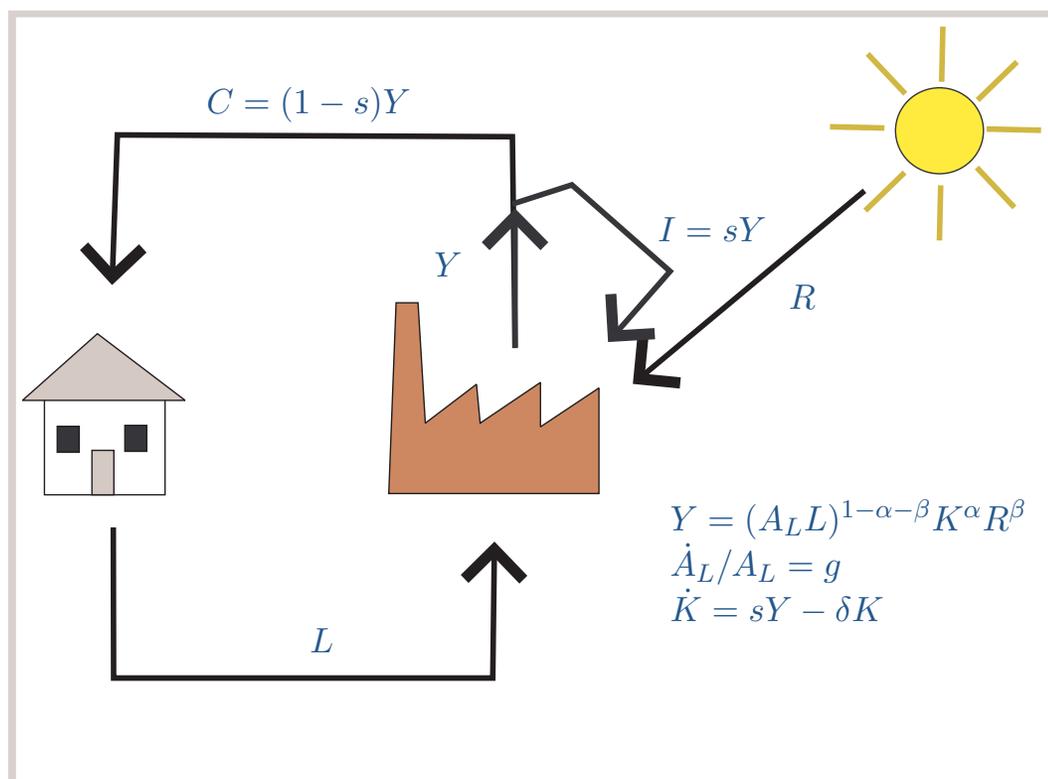


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**Neoclassical growth and  
nonrenewable resource supply:  
three simple cases**

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## Land (and 'flow renewables')

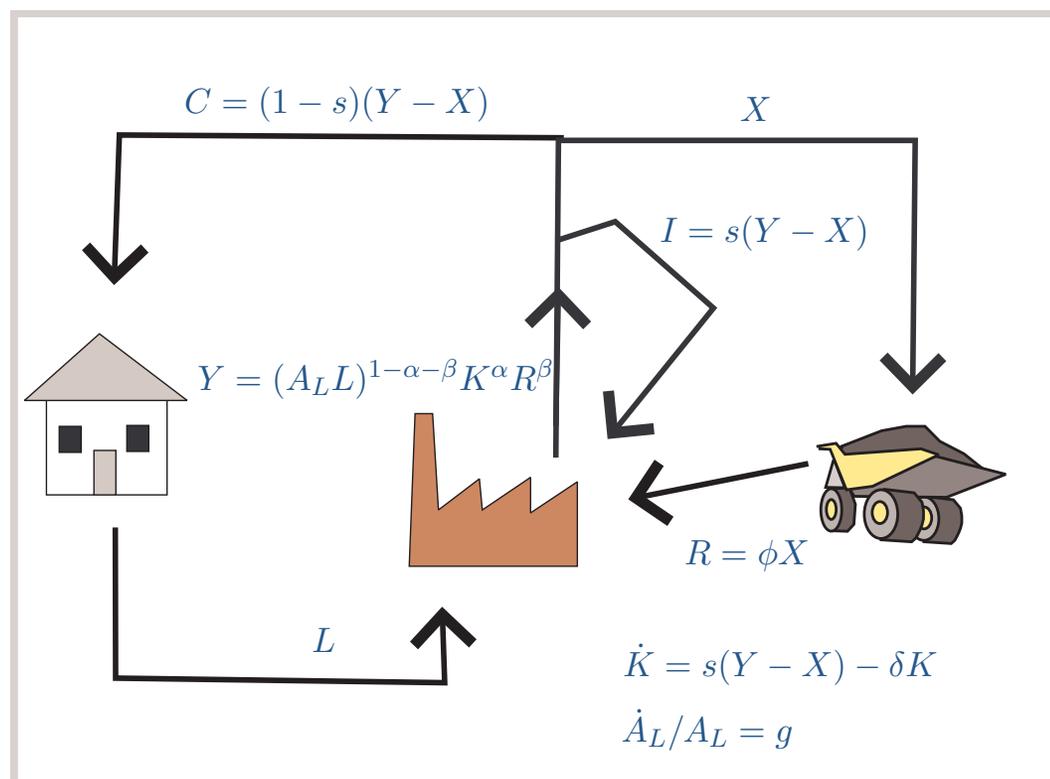


The economy with a fixed 'flow' resource.

Assume balanced growth. Characterize the b.g.p.

The representative final-good producer must hire labour, capital, and 'land'. What happens to the price of land over time?

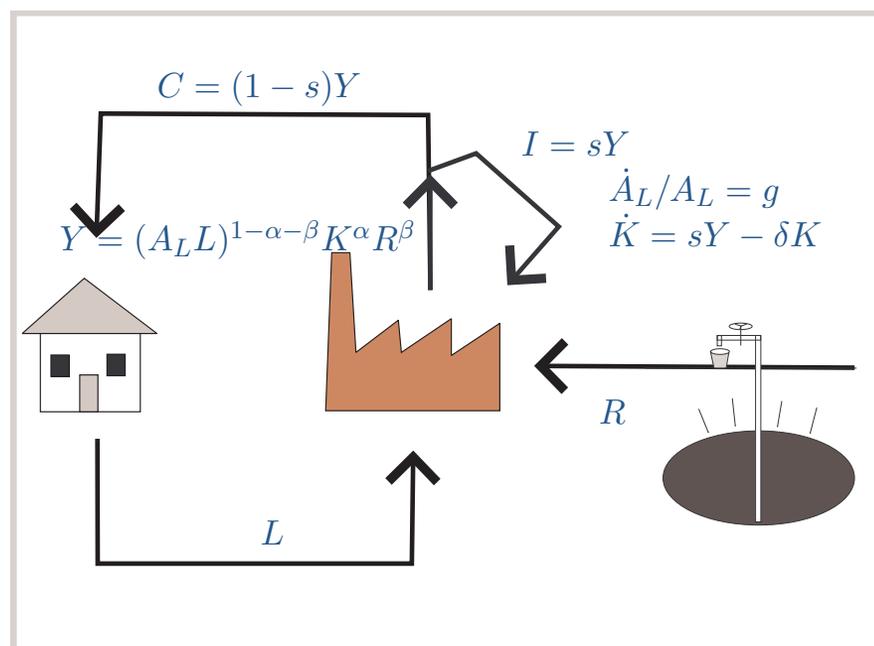
## An abundant resource, costly to extract



The economy with an unlimited resource, costly to extract.

Solve for growth rates on a b.g.p.!

## A limited resource, costless to extract (Hotelling/DHSS)



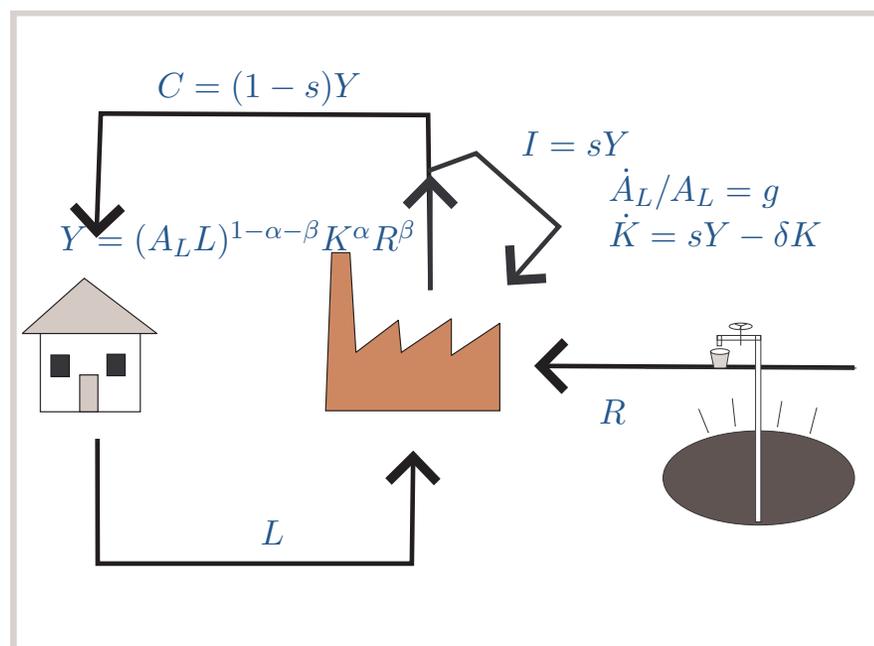
The economy with a limited resource, free to extract.

Assume a social planner who releases a resource flow  $R$  which declines over time at rate  $\theta$ . (So  $-\dot{R}/R = \theta$ .)

Solve for growth rates on the b.g.p.

What can we say about the initial rate of resource consumption?

## A limited resource, costless to extract (Hotelling/DHSS)



The economy with a limited resource, free to extract.

Assume perfect markets and symmetric equilibrium. What can we say about the resource price?

Solve for the b.g.p. in the market economy given that the interest rate is equal to the pure rate of time preference.

# The Hartwick rule

## The Hartwick rule

If we invest resource rents in capital, this guarantees sustainability?

NO!!!!!!!!!!

If, through investment in human-made capital, we can maintain constant overall capital stocks then we can sustain utility without technological progress.

But we can't do that because human-made capital depreciates.

## Historical data

## Historical data

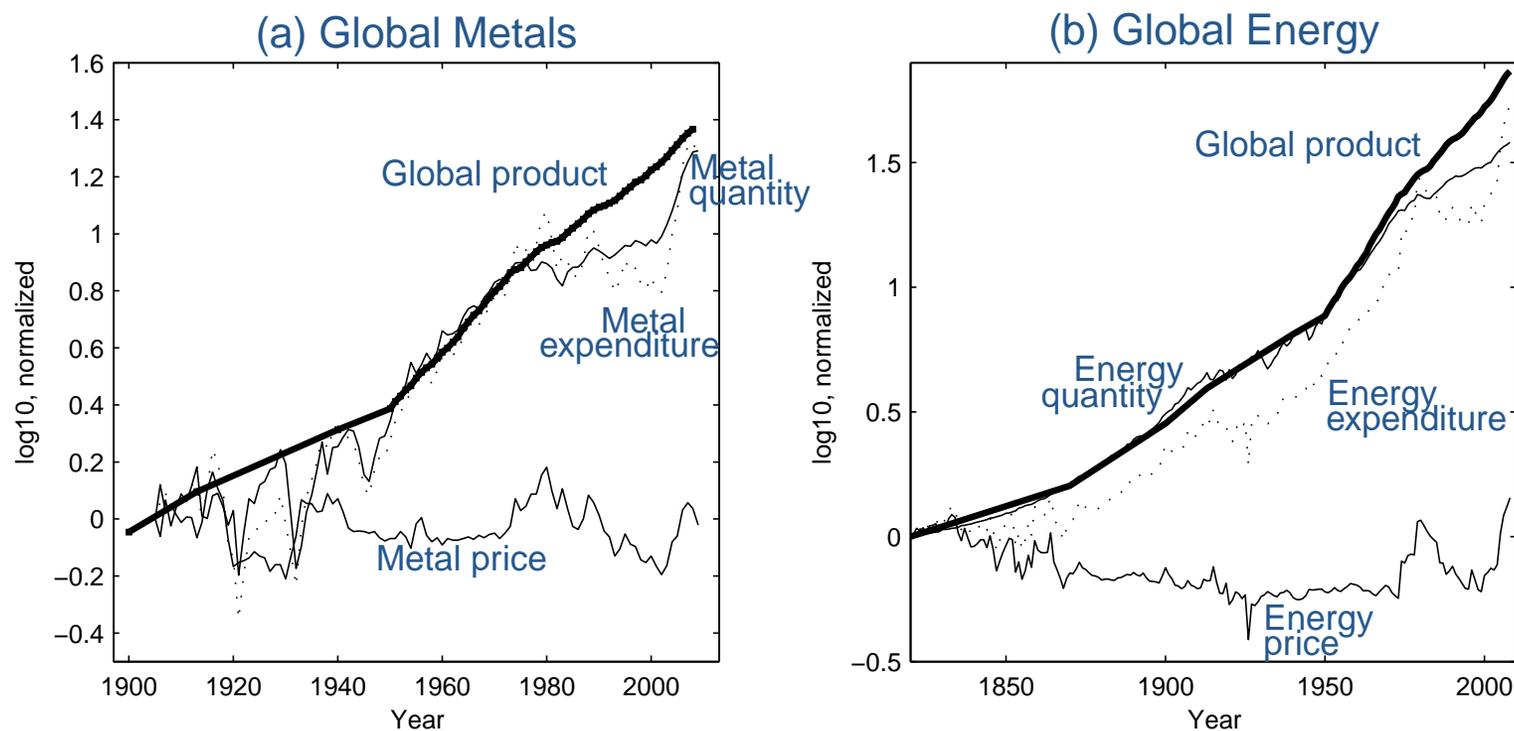


Figure 1: Long-run growth in consumption and prices, compared to growth in global product, for (a) Metals, and (b) Primary energy from combustion.

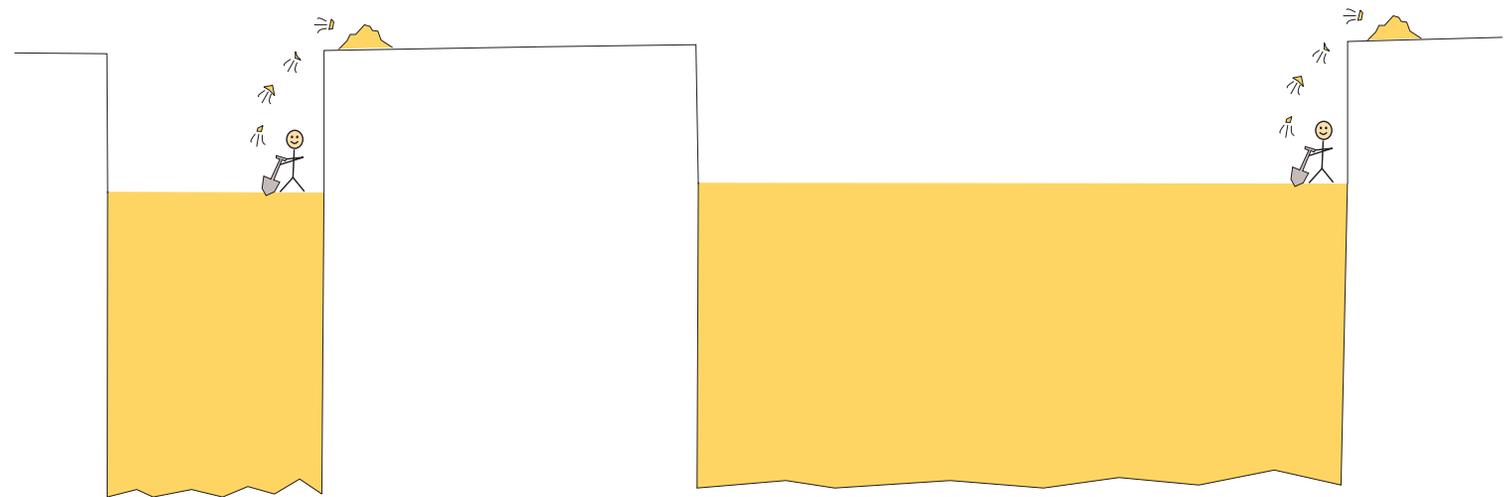


**Complex cases: limited  
resources, costly to extract**

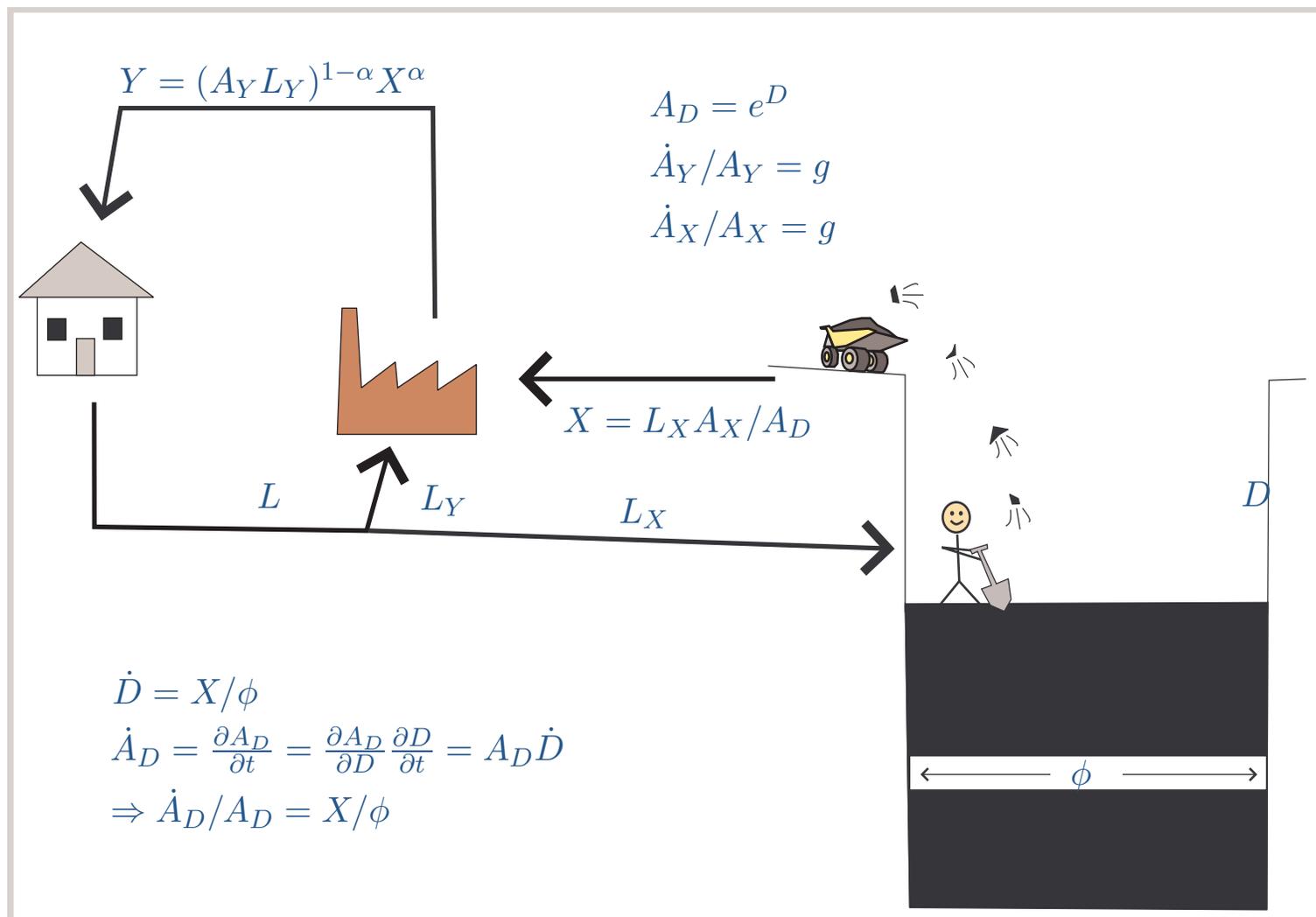
# Non-renewable resources in the long run

## Non-renewable resources in the long run

Consider the picture below, and with its help try to identify as many such factors as you can. Furthermore, categorize them according to whether they should make extraction costs rise or fall over time.

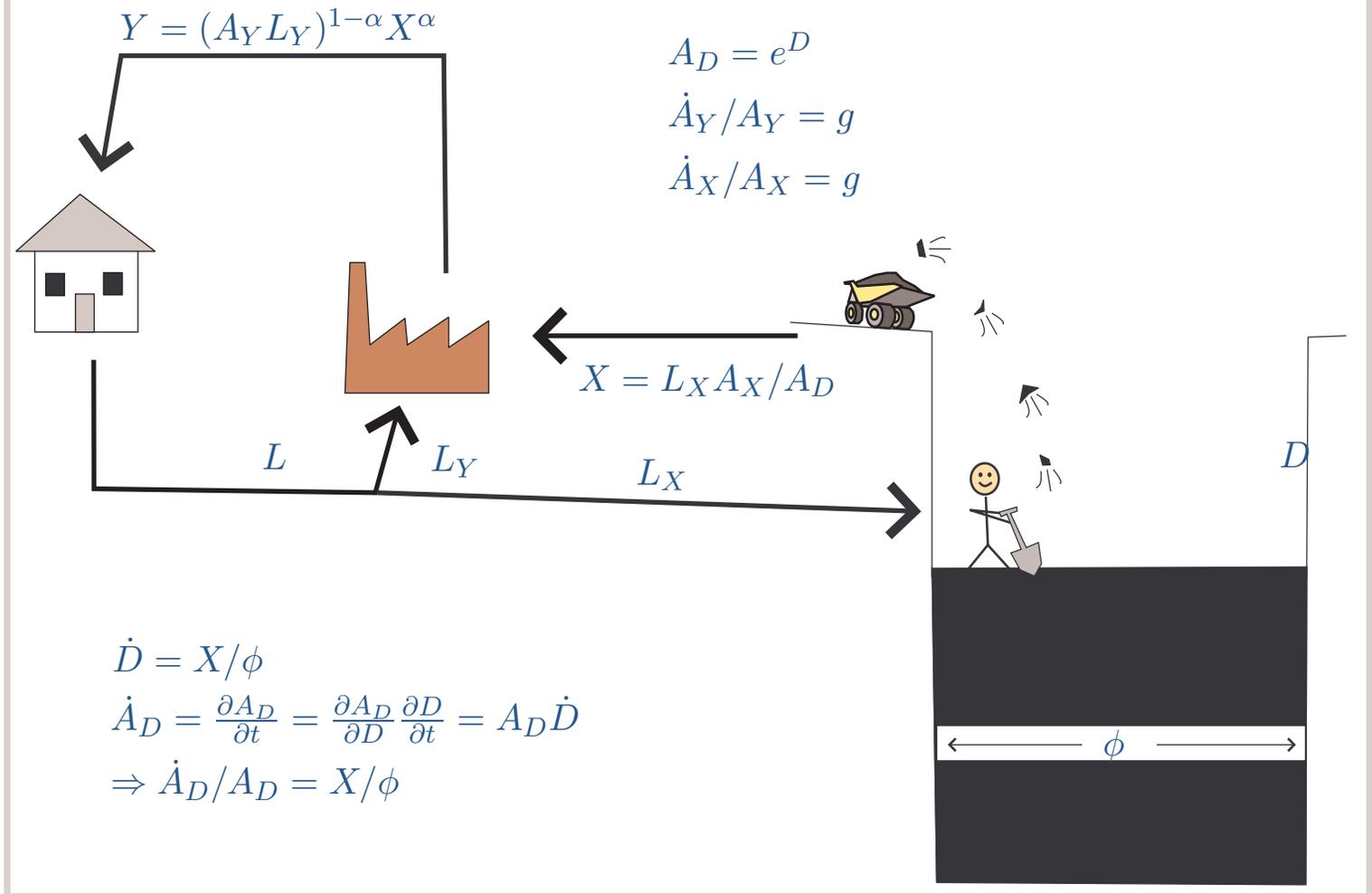


# An extraction model with inhomogeneous stocks



## An extraction model with inhomogeneous stocks

Now assume a *primitive* economy in which  $A_Y$  is very small. What happens?



## An extraction model with inhomogeneous stocks

Now assume a *primitive* economy in which  $A_Y$  is very small. What happens?

$$Y = (A_Y L_Y)^{1-\alpha} X^\alpha$$

$$A_D = e^D$$

$$\dot{A}_Y/A_Y = g$$

$$\dot{A}_X/A_X = g$$

$X$  small

$D$  and  $A_D$  constant

$$\dot{X}/X = g$$

$$\dot{Y}/Y = g$$

$$w_X X = \alpha Y$$

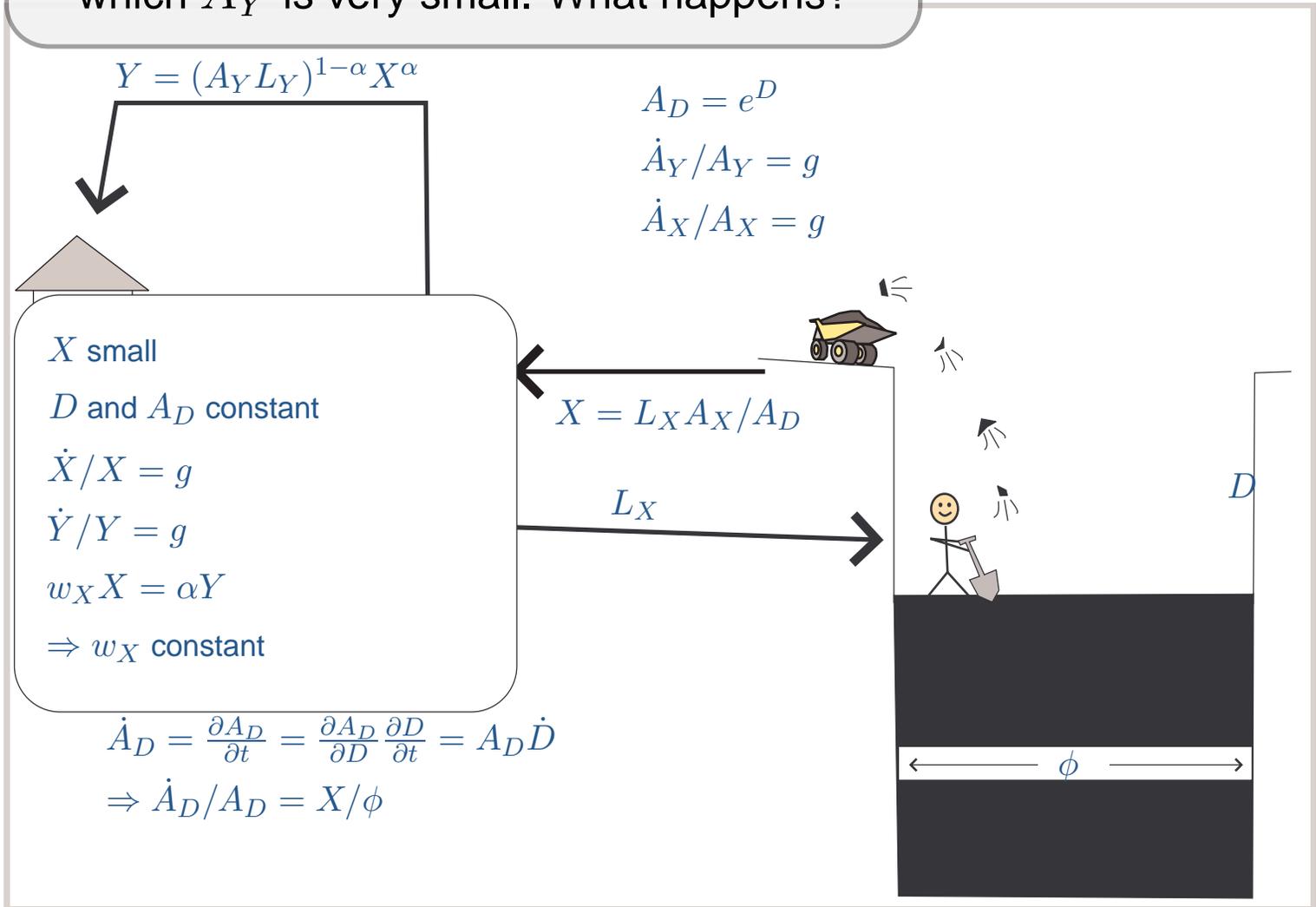
$\Rightarrow w_X$  constant

$$\dot{A}_D = \frac{\partial A_D}{\partial t} = \frac{\partial A_D}{\partial D} \frac{\partial D}{\partial t} = A_D \dot{D}$$

$$\Rightarrow \dot{A}_D/A_D = X/\phi$$

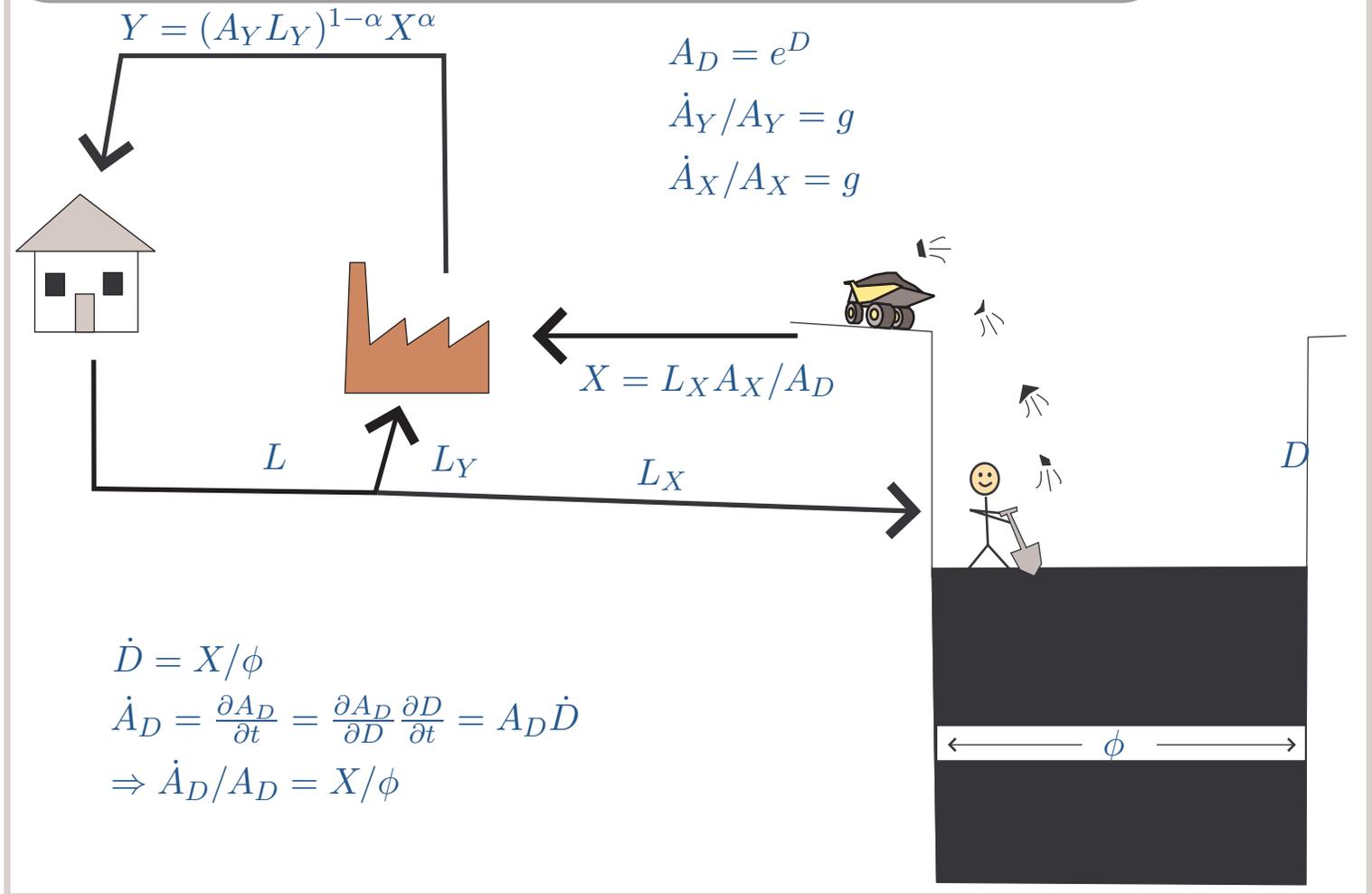
$$X = L_X A_X / A_D$$

$L_X$



## An extraction model with inhomogeneous stocks

Now assume a *balanced growth path* on which  $X$  is constant. Characterize the path.



## An extraction model with inhomogeneous stocks

Now assume a *balanced growth path* on which  $X$  is constant. Characterize the path.

$$Y = (A_Y L_Y)^{1-\alpha} X^\alpha$$

$$A_D = e^D$$

$$\dot{A}_Y/A_Y = g$$

$$\dot{A}_X/A_X = g$$

$$\dot{X}/X = 0$$

$$\dot{A}_D/A_D = \dot{A}_X/A_X = \theta_{AY}$$

$$X = g/\phi$$

$$\dot{Y}/Y = (1 - \alpha)\theta_{AY}$$

$$w_X X = \alpha Y$$

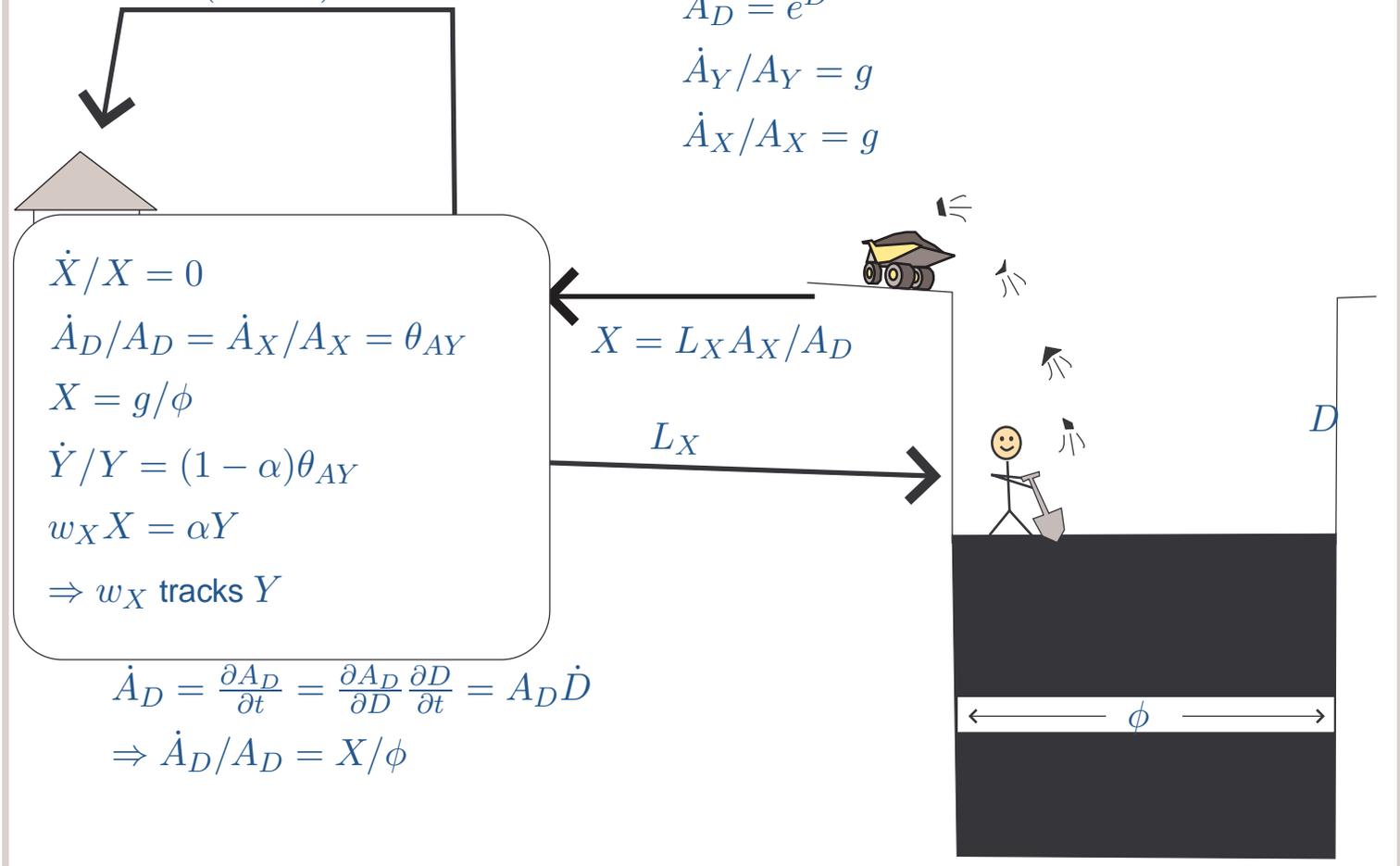
$$\Rightarrow w_X \text{ tracks } Y$$

$$\dot{A}_D = \frac{\partial A_D}{\partial t} = \frac{\partial A_D}{\partial D} \frac{\partial D}{\partial t} = A_D \dot{D}$$

$$\Rightarrow \dot{A}_D/A_D = X/\phi$$

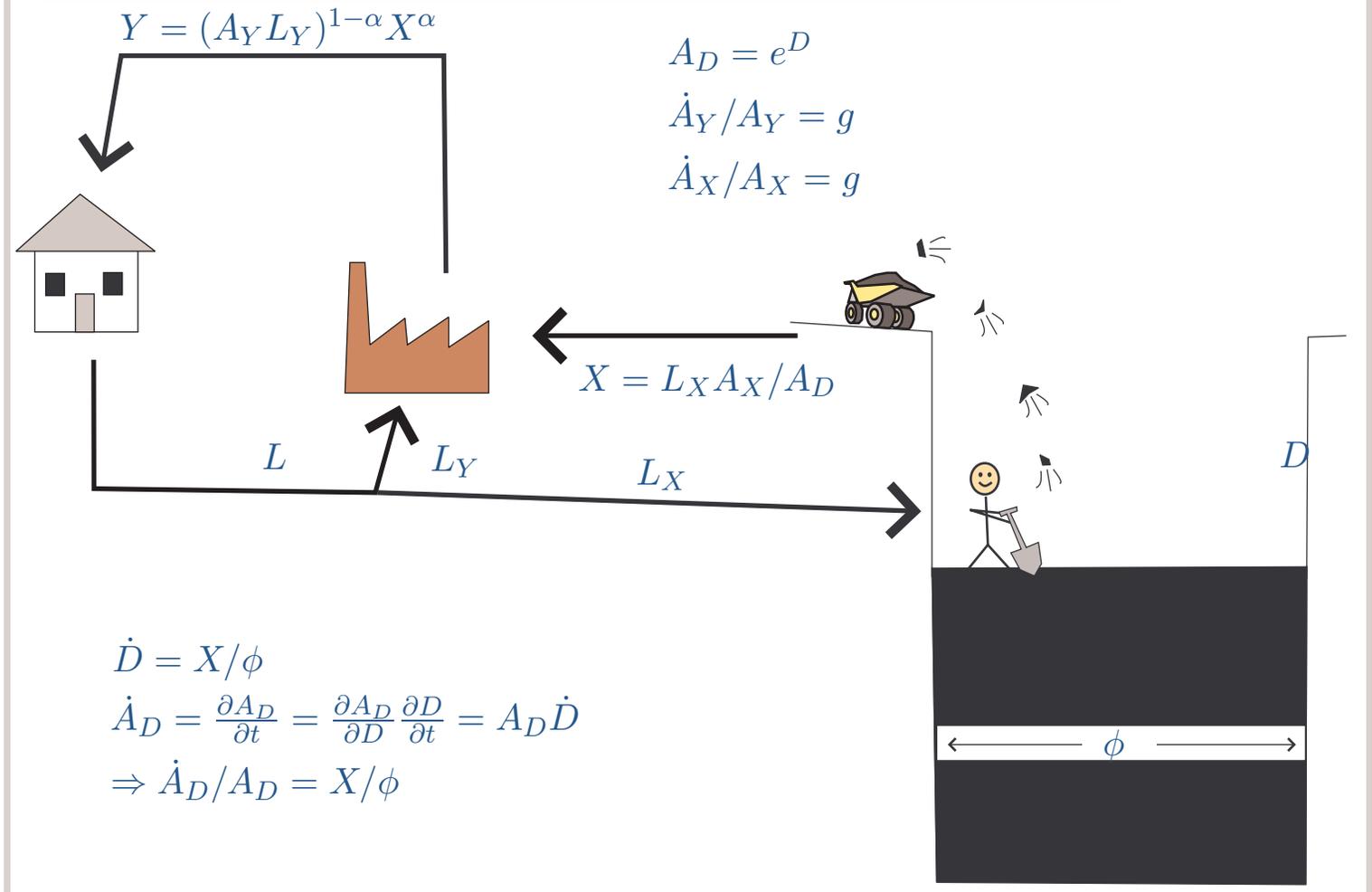
$$X = L_X A_X / A_D$$

$$L_X$$



## An extraction model with inhomogeneous stocks

Now assume a *Hotelling economy* in which the resource is running out. Characterize the path.



## An extraction model with inhomogeneous stocks

Now assume a *Hotelling economy* in which the resource is running out. Characterize the path.

$$Y = (A_Y L_Y)^{1-\alpha} X^\alpha$$

$$A_D = e^D$$

$$\dot{A}_Y/A_Y = g$$

$$\dot{A}_X/A_X = g$$

Depth constant

Extraction costs constant

Scarcity rent rises

⇒ pure scarcity in limit

$$\dot{w}_X/w_X \rightarrow \rho$$

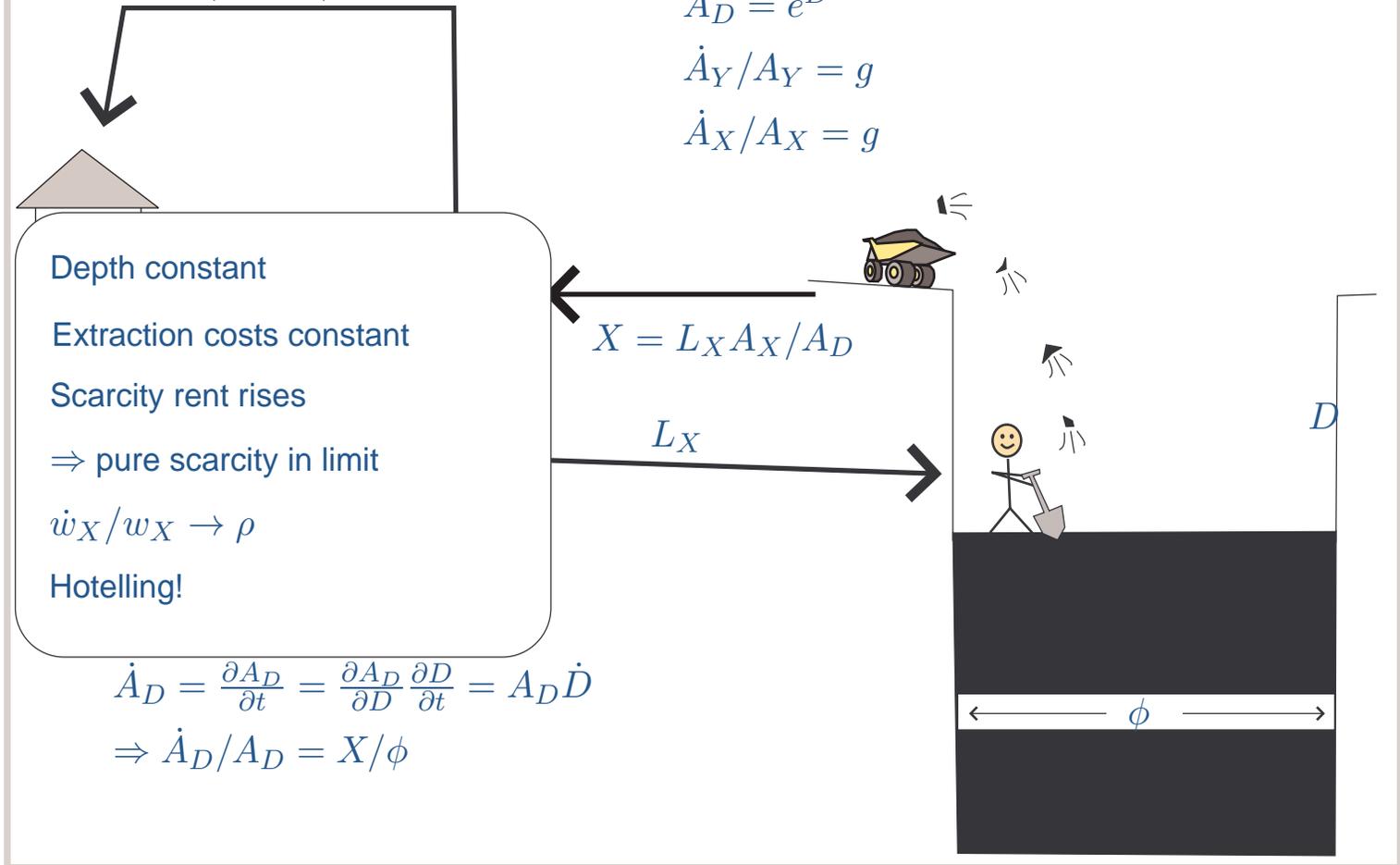
Hotelling!

$$\dot{A}_D = \frac{\partial A_D}{\partial t} = \frac{\partial A_D}{\partial D} \frac{\partial D}{\partial t} = A_D \dot{D}$$

$$\Rightarrow \dot{A}_D/A_D = X/\phi$$

$$X = L_X A_X / A_D$$

$L_X$



## An extraction model with inhomogeneous stocks

Empirical application?

# Lessons

## Lessons

We can solve the problems related to resource stocks, extraction costs, and predictions about the price.

I.e. we can solve the problems related to resource supply.

But what about resource demand, and the Cobb–Douglas production function?

## Lessons

Daly (1997):

In the Solow–Stiglitz variant, to make a cake we need not only the cook and his kitchen, but also some non-zero amount of flour, sugar, eggs, etc. This seems a great step forward until we realize that we could make our cake a thousand times bigger with no extra ingredients, if we simply would stir faster and use bigger bowls and ovens.

## Lessons

Solow (1973), 'Solow's three mechanisms':

1. Increase—through technological change—resource efficiency in production of one or more product categories;
2. Substitute on the consumption side away from product categories in which the production process is resource-intensive.
3. Increase—through technological change—the efficiency of an alternative (substitute) resource in production of one or more product categories.

## Lessons

Note that to explain the past using Solow's mechanisms we need to put them into reverse. Then we have three explanations for why resource and energy use has increased (tracking GDP):

1. resource-efficient technology has not been developed,
2. consumption patterns have shifted towards resource-intensive goods, and
3. there has not been substitution on the production side to alternative inputs, such as renewables.

To understand the mechanisms it is useful to look at past data. Then we can apply what we have learnt to predicting the future and designing policy.

## Lessons

Why the focus on capital substituting for resources?

