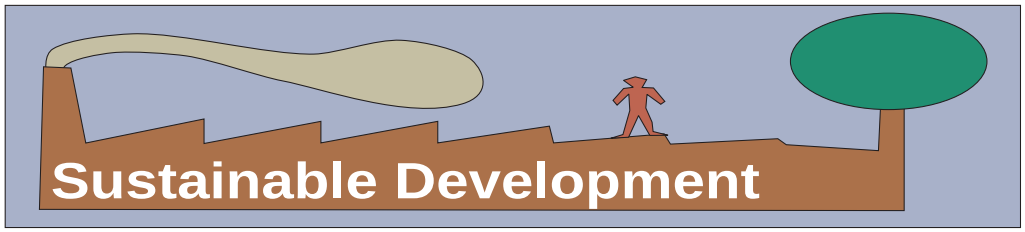
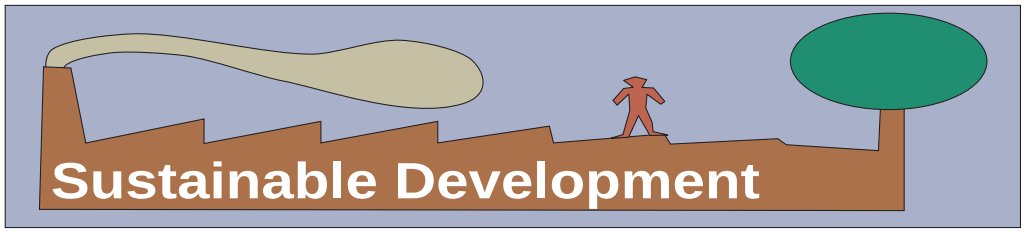



Chapter 3

# Post-Malthusian—industrial—growth

# The Solow and Ramsey models

# What is a production function?

# What is a production function?

Compare the following equations. Discuss similarities and differences.

$$2\,Fe_2O_3 + 3\,C \longrightarrow 4\,Fe + 3\,CO_2;$$

$$\Delta G = \Delta H - T\Delta S_{int};$$

$$Y_i = \min(A_L l_i, A_K k_i, A_R r_i).$$

# What is a production function?

Consider the following equations.

$$Y = A_L L,$$
$$Y = F(A_L L, A_K K),$$
$$Y = F(A_L L, A_K K, A_R R).$$

Suggest units for each of the variables.

What drives growth in $Y$? And $Y/L$?

# Neoclassical growth models

# Neoclassical growth models

The production function:

$$Y = F(A_K K, A_L L).$$

$$F'_K, F'_L \geq 0.$$

$$F''_K, F''_L \leq 0.$$

# Neoclassical growth models



Technology:

Nonrival. Nonexcludable. Exogenous.

# Neoclassical growth models



$$K_{t+1} = (1-\delta)K_t + sY_t,$$
$$K_{t+1} = (1-\delta)K_t + sF(K_t, L_t).$$

# Neoclassical growth models



Consider Figure 1. What are the values of $K$, $L$, $Y$, and $s$? And $\delta$?

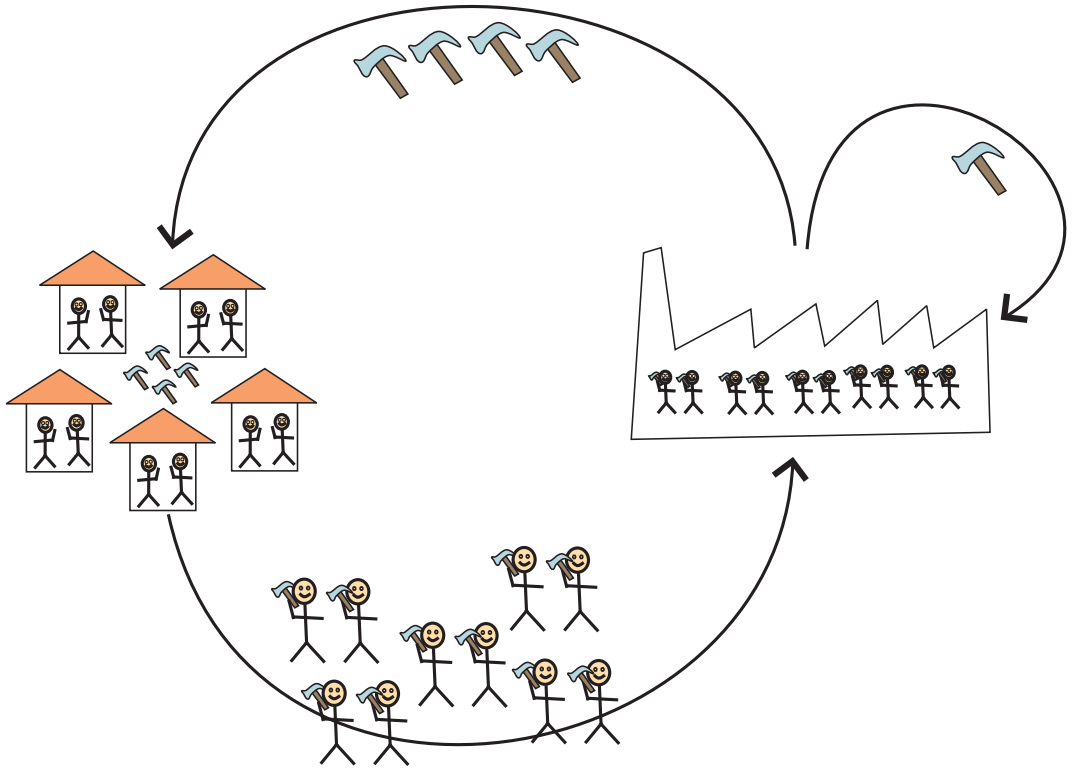


Figure 1: A Solovian economy in which hammers are the final good.

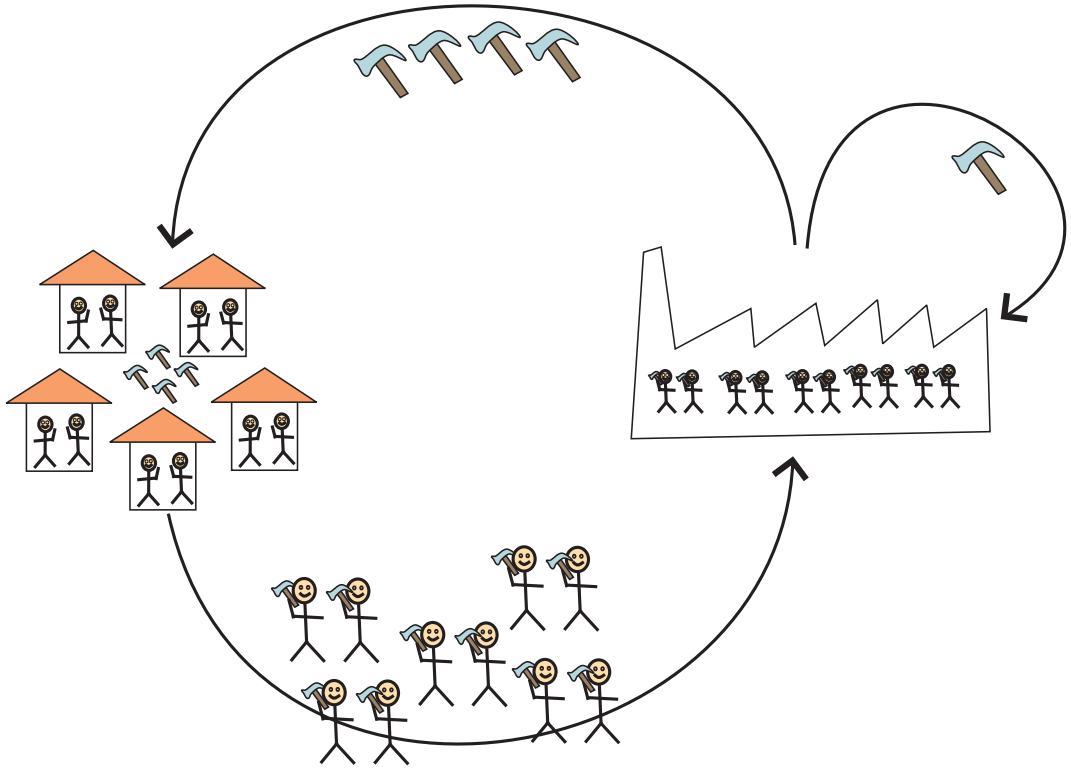
# Solow (with constant technology)

# Solow (with constant technology)

$$K_{t+1} = (1-\delta)K_t + sY_t.$$
$$Y = (A_L L)^{1-\alpha} K^{\alpha}.$$

Cobb–Douglas properties? Unit elasticity of substitution!

# Solow (with constant technology)



Solve the model when $\alpha = 1/3$ and $\delta = 0.1$.

Characterize the long-run steady state.

# Solow (with constant technology)

Discuss effects of different saving rates across countries.

And if there is a global capital market?

# Solow with exogenous growth in $A_L$

# Solow with exogenous growth in $A_L$

$$Y = (A_L L)^{1-\alpha} K^{\alpha}$$
$$\dot{A}_L/A_L = g_{A_L}$$
$$\dot{L}/L = n$$
$$\dot{K} = sY - \delta K.$$

On b.g.p., $\dot{K}/K$ constant, hence $sY/K - \delta$ constant, hence $Y/K$ constant, hence $(A_L L/K)^{1-\alpha}$ constant, and

$$\dot{K}/K = \dot{A}_L/A_L + \dot{L}/L.$$

$y = Y/L$. What is $\dot{y}/y$?

# Solow with exogenous growth in $A_L$



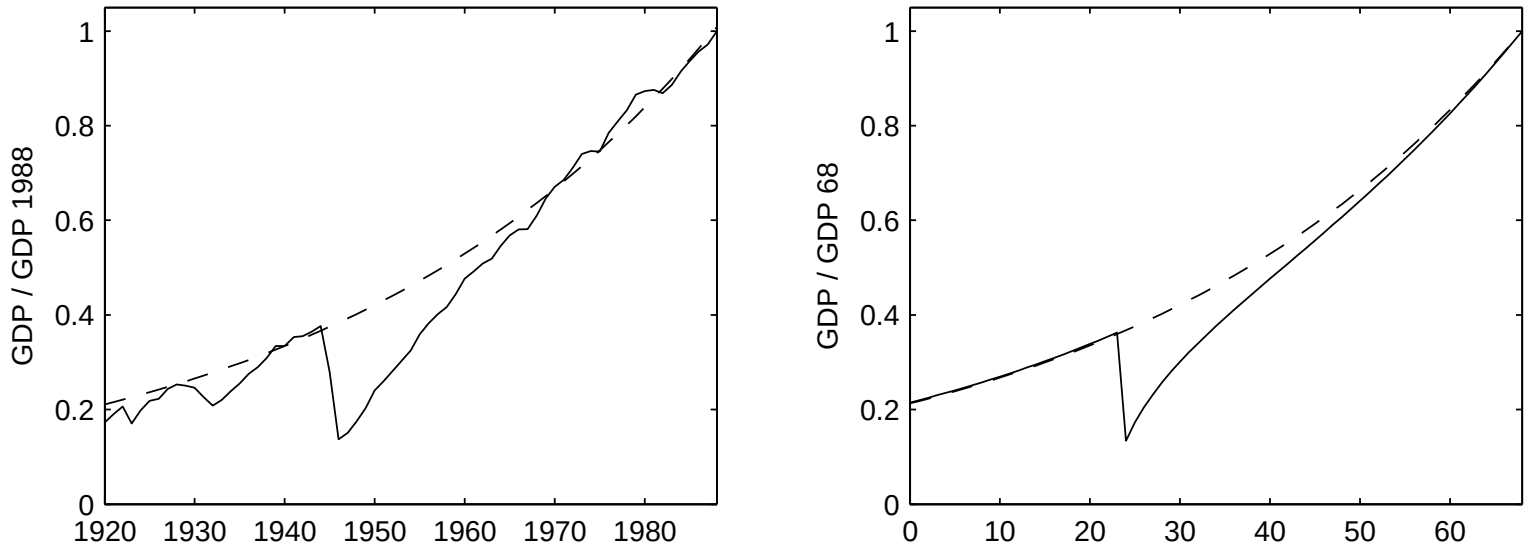



Figure 2: Testing the Solow model. Upper panel: German real GDP, compared to an estimated constant-growth trend (growth rate 2.3 percent per year). Lower panel: Model simulation assuming the same growth trend in labour productivity and parameters $\alpha = 0.3$, $\delta = 0.1$, $s = 0.2$, with a massive shock to capital (the stock is divided by a factor of 30 at the end of year 24).

# Solow with exogenous growth in $A_L$



What is the Solow model good for?

What drives growth in the Solow model?

What drives growth in real economies?

# Ramsey

# Ramsey

$$\max \sum_h U^h,$$

where

$$U^h = \sum_t u^h(c_t)\beta^t.$$

and

$$u = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

What is the Ramsey model good for?

# No capital

# No capital

$$Y = A_L L[K/(A_L L)]^{\alpha},$$

and note that in balanced growth $K$ grows at the same rate as $A_L L$, so $K/(A_L L)$ is constant, then it should be clear that in long-run analysis we lose little by simplifying the production function to

$$Y = A_L L.$$

The same conclusion applies when we include resources in the production function. Then we have (for instance)

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^{\beta} R^{\alpha} \\ &= (A_L L)^{1-\alpha} [K/(A_L L)]^{\beta} R^{\alpha}. \end{aligned}$$

What do we lose by dropping capital? Gain?

# Vintage growth models

# A simple model

# A simple model

One person per machine. Machines last for ten years, i.e. 1 period.

Year 2000, 100 people and 100 (new) machines. Of these, 80 people—each with a machine—work on the production of consumer goods, while 20 people—each with a machine—work on the manufacture of the next-generation machines.

100 new machines are ready in 2010, and they are 20 percent more productive than the old.

Period-1 wage: 100 crowns per period.

Interest rate per period of 10 years is 100 percent.

What is GDP in period 1? What is the labour share, and what is the capital share?

# A simple model

As above, but open economy far behind the technology frontier.

Produces and exports consumption goods, imports machines. Higher quality machines yield higher GDP per capita, but cost more.

Each quality step gives 20 percent higher GDP, and the machines to achieve it cost 20 percent more.

GDP per capita 10 000 USD per period, and for 2000 USD per capita the country can upgrade—in the next 10-year period—to machines that yield 12 000 USD per capita per period.

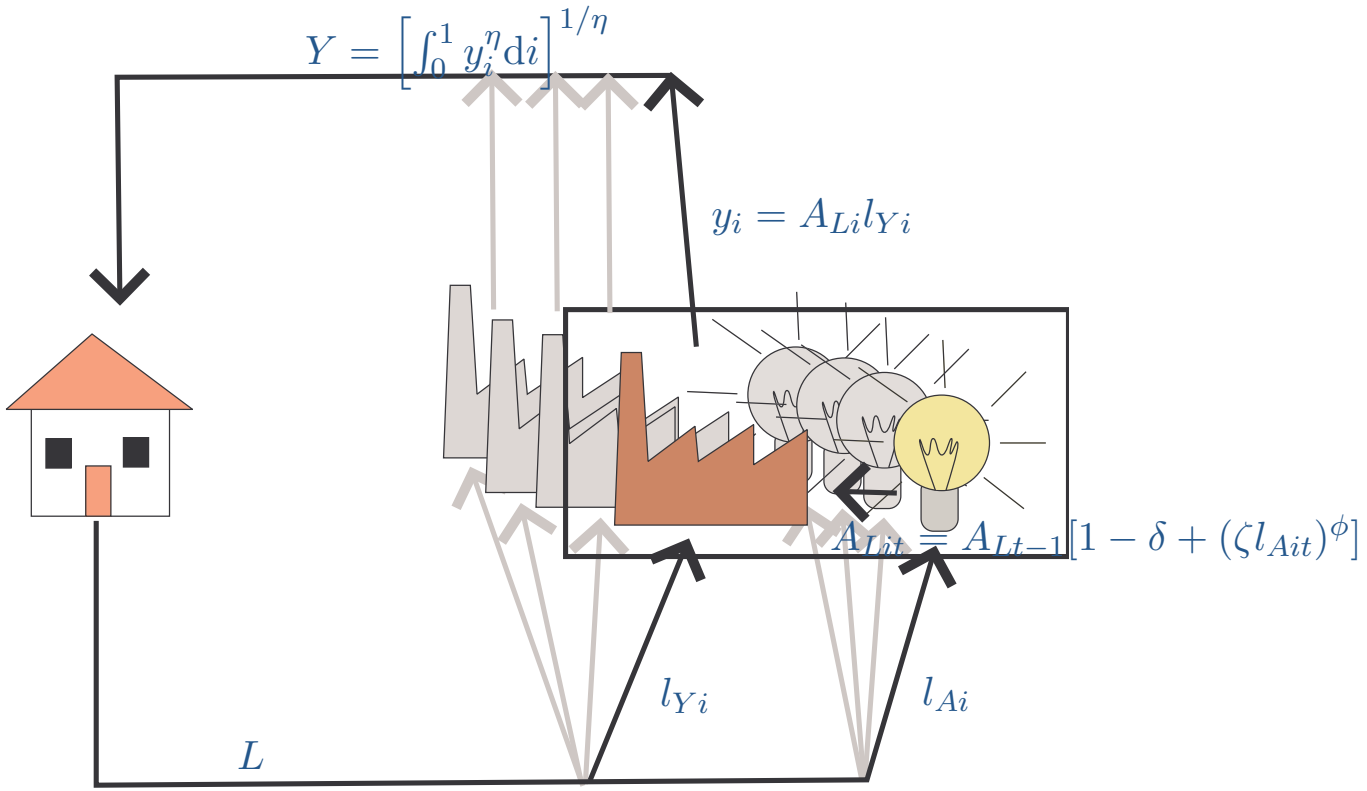
The country invests in new machines that are five rungs up the ladder compared to its current machines. What is investment as a proportion of GDP? Consumption? Growth rate?

# Endogenous growth

# Endogenous growth

$$A_{Lt+1} = A_{Lt}[1 - \delta + \zeta(\Omega L_t)^{\phi}],$$

# Endogenous growth




Single firm's Lagrangian? MR $= \eta p_i$. FOCs in $L_Y$, $L_A$, and $A_{Lit}$.

Investment in knowledge?

# Endogenous growth

We can take an ‘exogenous growth model’ in which workers devote a fixed proportion of their time to research for no obvious reason, and turn it into an endogenous growth model in which time spent on research is the result of competing firms solving optimization problems. For our purposes, the key use of the endogenous model will be in later chapters when we wish to analyse how firms choose between investment in alternative types of technology, such as ‘clean’ and ‘dirty’.