 Sustainable Development

Exercises: Brief answers[1]

1. Megafaunal extinction

Sorry, no written answers available for these questions yet.

2. Solow

2.1. The first three parts of this question are standard. Check notes and literature. Part (d) is a bit tricky though. Denote total factor productivity (TFP) by A, so that we can write $Y = AF(K,L)$ where F is CRS in K and L. And assume that A grows at a constant exogenous rate. Then if the savings rate is constant the economy converges to a balanced growth path on which both Y and K grow at the same rate as A. But to prove this is beyond the scope of the course.

2.2. (a) $A^{1-\alpha} = 0.5$, $\delta = 0.1$, $s = 0.2$.
  (b) 1.
  (c) The relative shares of K and L are fixed at $\alpha/(1-\alpha)$.
  (d) It makes no difference in the model.

2.3. (a) Technological progress.
  (b) What drives technological progress?
  (c) In long run, and when shifts between different types of product are known to be important. For instance in the case of resource or energy use.

3. DHSS

3.1. Check notes and literature for part (a), the answer being that I extract everything if the resource price is growing more slowly than money in the bank, and nothing if the resource price is growing faster than money in the bank; if the resource price grows at the same rate as money in the bank then I am indifferent between extracting now and extracting later. Part (b) is tricky. By setting up the Lagrangian and solving we find that

$$w_{rt+1}/w_{rt} = 1/\beta = 1+\rho,$$

where ρ is the interest rate per period. How is this related to the answer to (a)? After all, the price path and the interest rate are both exogenous, not affected by my extraction rate! The answer is that the equation is a necessary condition for the existence of a non-corner solution to my extraction problem. If it doesn't hold then the first-order condition cannot hold, implying that we must have a corner solution with either zero extraction or extracting everything in one go. And we still need the intuition from part (a) to complete the solution. To answer part (c) you must show that if the resource price is expected to rise 'too steeply' then no resource holders will sell, causing a jump up in the current price and thus reducing the expected rate of increase over time. Similarly, if the resource price is expected to rise 'too slowly' then all resource holders will try to sell their entire holdings, causing a jump down in the price and raising the expected rate of increase. In equilibrium the expected rate of increase must be 'just right', i.e. $w_{rt+1}/w_{rt} = 1+\rho$.

[1] Note that I just give very brief answers here for you to check against. Explanation and calculation is required in the exam!

3.2. Check lecture notes. Resources are available in a fixed, known, finite quantity, with zero extraction costs. Regarding their role in the production function, the elasticity of substitution between resources and other inputs is 1. If the resource share is small, this implies that changes in the quantity of resource inputs make very little difference to the quantity of production. Perhaps resources could be some kind of energy input; they are necessary, but resource use per unit of production can approach zero. They seem to be, in some sense, 'waste', i.e. not incorporated in the final product.

3.3. For part (a), note that if resource use is declining at a fixed rate θ then we can write

$$R_t = R_0 e^{-\theta t}.$$

Integrate this between 0 and ∞ to show that

$$\theta = R_0/S$$

if the resource is to be asymptotically exhausted. For part (b) note that we can write $\dot{K} = sY$, hence $\dot{K}/K = sY/K$. For the final part, the answer is

$$g_Y = \frac{1-\alpha-\beta}{1-\alpha} g_A - \frac{\beta}{1-\alpha}\theta.$$

3.4. (a) Write $Y - C = sY$ on balanced growth, and then find expression for $\dot{K}/K$ ...

(b) Differentiate the production function w.r.t. time and then use the answer to part (a) to show that

$$\dot{Y}/Y = \frac{1-\alpha-\beta}{1-\alpha}(g_A + n).$$

Finally, since $Y = yL$ then $\dot{Y}/Y = \dot{y}/y + n$, hence

$$\dot{y}/y = \frac{1-\alpha-\beta}{1-\alpha} g_A - \frac{\beta}{1-\alpha} n.$$

(c) If y is constant then

$$\frac{1-\alpha-\beta}{1-\alpha} g_A = \frac{\beta}{1-\alpha} n,$$

hence

$$n = \frac{1-\alpha-\beta}{\beta} g_A.$$

This implies that—given reasonable values for the parameters—a slow rate of technological progress allows a rather rapid rate of population growth.

(d) To enrich the model further we could make growth in A_L endogenous (endogenous growth), and also the saving rate s (Ramsey).

(e) When population growth stops then we have

$$g_y = g_Y = \frac{1-\alpha-\beta}{1-\alpha} g_A.$$

(f) The fact that there is a fixed quantity of land puts a small penalty on the growth rate that can be achieved, compared to a situation in which land is unlimited so $\beta = 0$.

3.5. Here we aren't told anything about resource prices. Therefore you should state what you assume. In the spirit of the 'Limits' model we assume that they are free. So we have growth based on increasing labour productivity, and resources get 'sucked in' due to the Leontief production function and the fact that A_R either doesn't grow at all, or (when it grows) the growth rate declines over time. At some point the stock runs out and the economy crashes. Concerning the views of the critics, refer to the literature.

3.6. No. 'Their' model is based on capital accumulation and does not allow us to better understand or test the strength of the three mechanisms given.

3.7. Check lecture notes. The key ones are 'depth', technology and wages. The effects of the latter two tend to cancel each other out.

3.8. (a) $p_x = \alpha y/x$.

(b) The expressions are:

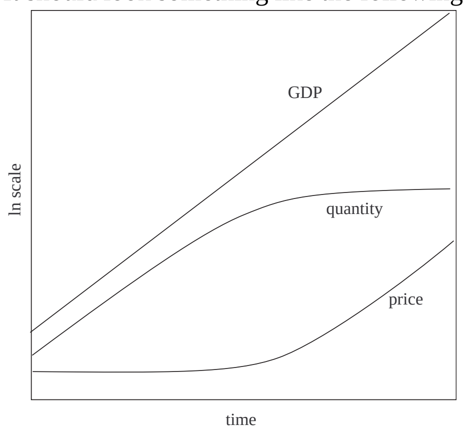
$$\dot{x}/x = \theta_{ax} - \theta_{bx},$$
$$\dot{p}_x/p_x = \dot{y}/y - \dot{x}/x,$$
$$\text{and} \qquad \dot{y}/y = (1-\alpha)\theta_{ax} + \alpha\dot{x}/x.$$

(c) In the primitive economy b_x is constant because the extraction rate is very low. Furthermore we are told that $\theta_{ax} = \theta_{ay}$. So we have

$$\dot{x}/x = \theta_{ax},$$
$$\dot{p}_x/p_x = 0,$$
$$\text{and} \qquad \dot{y}/y = \theta_{ax}.$$

Over time the extraction rate increases, and b_x can no longer be assumed to be constant. That is, the economy leaves the initial b.g.p.

(d) It should look something like the following figure.



(e) When the resource is close to exhaustion and there are no substitutes then extraction costs become irrelevant and scarcity takes over. That is, the price rises at the discount rate, and resource use declines exponentially towards zero:

$$\dot{p}_x/p_x = \rho.$$

4. Resource demand and pollution: Solow's three mechanisms

4.1. (a) $Y = [(A_L L)^\varepsilon + (A_R R)^\varepsilon]^{1/\varepsilon}$. The parameter ε must be negative.

(b) Labour: Workers.
Resource: Tons / day.
A_L: Widgets/(worker·day).
A_R: Widgets/ton.

(c) Relative factor shares: $w_L L/(w_R R) = [A_L L/(A_R R)]^\varepsilon$.

4.2. (a) 91 houses per week total, 0.91 per capita.

(b) Hmm. Now it's 909. Seems like rather a lot ...

(c) In the first case $w_a = 0.826$, $w_b = 0.826$, and $w_a Q_a/(w_b Q_b) = 10$.
In the second case $w_a = 0.826$, $w_b = 82.646$, and $w_a Q_a/(w_b Q_b) = 0.1$.

(d) The rate of housebuilding doubles, with input quantities and relative prices constant. So K_a and K_b are both doubled.

4.3. The Lagrangian should look something like this:

$$\begin{aligned}\mathcal{L} = & p_{yt}[(k_{lt}q_{lt})^\varepsilon + (k_{rt}q_{rt})^\varepsilon]^{1/\varepsilon} - w_z(z_{lt} + z_{rt}) - (w_{lt}q_{lt} + w_{rt}q_{rt}) \\ & - \lambda_{lt}(k_{lt} - \zeta_l k_{lt-1} z_{lt}^\phi) - \lambda_{rt}(k_{rt} - \zeta_r k_{rt-1} z_{rt}^\phi).\end{aligned}$$

You should find that $z_l/z_r = w_l q_l/(w_r q_r)$. Since $\varepsilon < 0$ there is a stable b.g.p., along which the factor shares are such that $z_l/z_r = w_l q_l/(w_r q_r) = (\zeta_r/\zeta_l)^{1/\phi}$. So sustainability under resource constraints is no problem, a bit like in DHSS. In DHSS you get sustainability by investing in capital, here you get it by investing in knowledge. In both cases very optimistic assumptions are made about the long-run returns to such investment.

4.4. (a) The first thing to get a grip on here is the rebound effect in general. Assume that energy efficiency in production of good 1 increases by 1 percent. If there is no rebound (i.e. no reallocation of labour between goods 1 and 2) then energy use in production of good 1 will go down by 1 percent, whereas energy use in production of good 2 will be unchanged. The percentage drop in total energy use R will then be

$$\frac{r_1}{R} = \frac{r_1}{r_1 + r_2}.$$

So if $\eta_r = -r_1/(r_1 + r_2)$ then there is no rebound effect.
Next note that in cases (i)–(iv) we have the following: (i) no rebound; (ii) no rebound; (iii) negative rebound; (iv) positive rebound. The reasons are as follows: (i) no reallocation of labour; (ii) there is a reallocation of labour, but it makes no difference as the products are equally energy-intensive; (iii) reallocation towards low-energy products, therefore negative rebound; (iv) reallocation towards energy-intensive products, therefore positive rebound.

(b) Since we know that many products use energy, and that energy typically accounts for anything from 1 to 15 percent of factor costs, we know that rebound effects of energy-saving technological progress may commonly be either negative or positive, with progress on product categories which are already of low energy intensity leading to negative rebound. On the other hand, progress on product categories of high energy intensity may lead to more rebound. But note of course that the latter type of product may account for much more energy use.
Regarding policy, given the complex results described above, a careful study of each case is needed to judge the overall effects of policy to boost energy-efficiency in a given product or set of products.

4.5. (a) You should find that unit costs, and hence the price p_y, is given by

$$p_y = \frac{w_l}{A_l} + \frac{w_e}{A_e}.$$

(b) From the demand function and $Y = A_e E$ we have

$$E = \alpha p_y^{-\eta} / A_e = \frac{\alpha}{A_e} \left(\frac{w_l}{A_l} + \frac{w_e}{A_e} \right)^{-\eta}$$

(c) For simplicity let's assume that $\eta = 1$, as given at the start of the question. Then we have

$$E = \alpha \left(\frac{w_l A_e}{A_l} + w_e \right)^{-1},$$

and the elasticities are

$$\eta_{Ae} = -\frac{\frac{w_l A_e}{A_l}}{\frac{w_l A_e}{A_l} + w_e} \quad \text{and} \quad \eta_{we} = -\frac{w_e}{\frac{w_l A_e}{A_l} + w_e}$$

(d) The elasticities are $-5/6$ and $-1/6$. So energy demand is more sensitive to energy efficiency than it is to the price of energy. (Why is this, intuitively? It has to do with the factor share of energy ...) On the other hand, it may be easy for a regulator to raise the price of energy, but very hard to raise energy efficiency. And, of course, a rise in the energy price may be required on efficiency grounds (i.e. a Pigovian tax).

(e) In the long run demand elasticity is likely to be greater, as effects build up over time, thus reducing the effect energy-efficiency improvements (because of greater rebound) and increasing the effect of energy-price increases. Furthermore, an energy-price increase should lead to energy-efficiency increases over time.

4.6. For (a), the production functions from question 3.3 would do fine, with adjusted notation. For (b), note firstly that D is cheaper, hence it is preferred to C. Furthermore, its knowledge is also easier to build through research. Hence D has an unambiguous advantage over C, and (if prices are exogenous and fixed) only D will be used. (c) The regulator, in order to encourage a switch from D to C, must make a one-time intervention to make C more attractive than D; Once firms prefer C, investment will go that way, and C will remain dominant.

4.7. (a) $p_l L/(p_r R) = \alpha/(1-\alpha)$.
(b) $w_l L/(w_r R) = \alpha/(1-\alpha)$.
(c) $w_c C/(w_d D) = (C/D)^{\varepsilon} = (w_c/w_d)^{-\varepsilon/(1-\varepsilon)}$.
(d) C enters the market and takes a share in accordance with its relative price (or quantity if that is the exogenous variable.
(e) Fits reasonably well, although the instant adaptation is of course oversimplified. At least, that's my story.
(f) Short-run policy to favour renewables will never be enough. (Compare to Acemoglu *et al* 2012.)

4.8. Examples of possible functions ...
(a) $Y = [(k_l q_l)^{\varepsilon} + (k_r q_r)^{\varepsilon}]^{1/\varepsilon}$.
(b) $Y = \min\{k_l q_l, [(k_c q_c)^{\varepsilon} + (k_d q_d)^{\varepsilon}]^{1/\varepsilon}\}$.
(c) $Y = (k_l q_l)^{\alpha} Y_R^{1-\alpha}$; $Y_R = [(k_c q_c)^{\varepsilon} + (k_d q_d)^{\varepsilon}]^{1/\varepsilon}$.

5. Pollution

5.1. The difference is caused by a number of factors. On the 'resource' side it is simple: resource prices have remained remarkably constant, as increasing productivity of extraction inputs is approximately matched by increases in the prices of those inputs (which is not surprising since the productivity of those inputs also increases in other parts of the economy) while increasing depth (or decreasing quality) of the marginal resource deposits has had little effect on the price.

The 'pollution' side is more complex, because the costs of emitting pollution are twofold: firstly, the costs of buying the input that gives rise to the polluting emissions, and second the cost of paying for Pigovian taxes (or costs imposed due to other regulatory instruments; for instance a ban on emitting a pollutant corresponds to an infinite unit cost of emissions). The former cost is constant, whereas the latter rises as a function of GDP (the higher is GDP, the higher is the willingness to pay to avoid suffering a given level of pollution). As long as the latter cost is small relative to the former, polluting emissions will behave like resource use. But when the latter becomes large compared to the former the overall cost of polluting starts to rise, and polluting emissions are braked.

This explains why polluting emissions might level off, but why would they fall so steeply? This will happen if the price rise makes some other production process (which uses a different input, or involves end-of-pipe cleanup of the pollutant) becomes cheaper than the polluting process. At this point polluting emissions fall rapidly to a new level, which is determined by how clean (or polluting) the new process is.

5.2. (a,b) The marginal costs are input costs plus damages:

$$MC_1 = w_1 + \psi(A_L L)^{1-\alpha} R^{\alpha};$$
$$MC_2 = w_1(1+\gamma).$$

(c) The marginal benefits are identical for the two inputs:

$$MB_1 = MB_2 = \alpha Y/R.$$

(d) The condition is

$$w_1 \gamma/\psi = (A_L L)^{1-\alpha} R^{\alpha},$$

and it implies that as A_L and L grow (also causing X_1 and hence R to grow) there comes a point when it is better to use input X_2 instead of X_1.
(e) If the economy is optimally regulated then R will initially be produced using X_1, and X_1 (and also pollution flows) will grow at a high rate (close to the overall growth rate). As GDP increases, pollution damages become significant, and brake the growth in X_1 somewhat (this could be through the use of a Pigovian tax, for instance). Then, when the condition above is fulfilled, the tax becomes so high that the economy switches totally to input X_2. (Alternatively, input X_1 is banned.)
(f) If there are multiple alternative inputs then this process could be repeated many times.
(g) If the alternatives were imperfect substitutes then the transition from one input to the other would be more gradual (but still potentially quite abrupt).

6. Labour supply and sustainable development

6.1. (a) Labour h is the same as $1-l$ (we are told this). And if the wage is w then consumption is $wh = w(1-l)$. And w is of course linked to productivity A.
(b) This is 'status', the ratio of own consumption to average consumption.
(c) The three arguments—consumption, status, and leisure—are poorly substitutable for one another. If the price of one of them falls, we will spend less money on it rather than more. This implies $\varepsilon < 1$.
(d) The input $c_i/\bar{c}$ is always equal to 1 in symmetric equilibrium. When productivity is very low both consumption and leisure will be very how, hence $c_i/\bar{c}$ is greater than the other two. As productivity increases households prioritize the inputs in short supply, i.e. consumption and leisure. They consume more, but also reduce their labour hours.
(e) When productivity is high both consumption and leisure can grow without bound, while the status term is locked at 1. Hence it falls below the others, and households give it increasing attention, supplying more labour in a doomed attempt to raise $c_i/\bar{c}$. Hence labour supply stops falling, and consumption continues to rise.
(f) A social planner would let labour supply approach zero as productivity approached infinity.
(g) Maybe we need policy instruments to help us chill out more, helping to solve the consumption externality.

6.2. The externality could also affect patterns of consumption. We should tend to focus more and more on goods that give status. But what goods give status? If it is energy-intensive goods such as big houses, powerful cars, and international travel then we are in trouble. But maybe this will change over time...

6.3. (a) If I work hard this pushes up $\bar{c}$ for everyone else, lowering their utility. My status rises, their's falls.
(b) Because of the above, everyone works hard (sacrificing leisure) playing the zero-sum status game. If they could all agree to chill out more they would all be happier.
(c) The regulator could impose a tax on income (or consumption) to reduce labour supply. See below for how to work out the level of the tax.
(d) The extended model (previous questions) is more relevant because here we see that the strength of the status effects increases in a growing economy, causing increasing problems. Potentially we can also explain consumption patterns with this kind of model.

To solve for optimal policy, first take the tax as exogenous and work out how much each household chooses to work. To do so, substitute into the utility function to yield

$$u = \left[l_i(1-\tau) + \bar{l}\tau\right]^{\alpha} (R - l_i)^{1-\alpha} / \bar{c}^{\alpha_2}.$$

Take the first-order condition in l_i and solve to show that

$$l_i = \alpha R - (1-\alpha)\bar{l}\tau/(1-\tau).$$

So when income tax is zero $l_i = \alpha R$: as labour income dominates the utility function (α high) households devote more of their time to labour and less to leisure.

Now assume a symmetric equilibrium such that average labour $\bar{l}$ is equal to the labour supplied by household i, l_i. Inserting this into the above result we have

$$\text{(a)} \qquad l_i = \bar{l} = \frac{\alpha R}{1 + (1-\alpha)\tau/(1-\tau)}.$$

Now the question for a regulator is, what level of tax τ maximizes utility for households? Economic theory tells us that if markets are perfect then the optimal tax should be zero, hence $l_i = \alpha R$. But if there is a consumption externality—i.e. if $\alpha_2 > 0$—then this no longer holds.

To solve the problem, we insert the expression for l_i as a function of τ into the utility function—noting that in symmetric equilibrium $c_i = \bar{c}$ and (as already stated) $l_i = \bar{l}$—to obtain

$$u = \left[\frac{\alpha R}{1 + (1-\alpha)\tau/(1-\tau)}\right]^{\alpha_1} \left[R - \frac{\alpha R}{1 + (1-\alpha)\tau/(1-\tau)}\right]^{1-\alpha}.$$

Simplify to obtain

$$u = R^{1-\alpha_2} \alpha^{\alpha_1} \omega^{-(1-\alpha_2)} (\omega - \alpha)^{1-\alpha},$$

where $$\omega = 1 + (1-\alpha)\tau/(1-\tau).$$

Take the first-order condition in ω to solve for the optimal ω, and then use the definition of ω to solve for the optimal tax:

(b) $$\tau = \frac{\alpha_2}{\alpha_1 + \alpha_2}.$$

So, the stronger the weight of 'conspicuous consumption' in utility, the more labour income should be taxed.

How big is the effect? Assuming conspicuous consumption has equal weight to consumption in utility then labour income should be taxed at 50 percent. The effect of the tax is to reduce labour supply by a factor ω. And if leisure has 50 percent weight in utility (implying that $\alpha = 0.5$ so in laissez-faire the individuals would work 8 hours and have 8 hours of leisure time, assuming that 8 hours are needed for sleep) then $\omega = 1.5$, so labour supply is reduced by one third.