Economic Growth and Sustainable Development, NA0167.

Examination, January 2021, suggested answers

1. You are given the following two models.

   - Model 1 (a variation on the 'limits to growth' model).

   $$Y_t = \min\{A_{Lt}L_t, A_{Rt}R_t\};$$
   $$\dot{A}_L/A_L = g;$$
   $$\int_0^\infty R_t \mathrm{d}t \leq S_0.$$

     Labour L is fixed, and hired on perfect markets. The resource R is costless to extract and is of 'open access' character, i.e. no individual or group has property rights over the resource (and it is not storable after extraction). A_R is constant.

   - Model 2 (a variation of the DHSS model with a resource in infinite supply but costly to extract, and competitive markets).

   $$Y = (A_L L)^{1-\alpha-\beta}K^\alpha (A_R R)^\beta;$$
   $$\dot{A}_L/A_L = g;$$
   $$\dot{K} = s(Y - X) - \delta K;$$
   $$C = (1 - s)(Y - X)$$
   $$R = \phi X.$$

     Again, A_R is constant.

   (a) i. Consider Model 1, and explain carefully (mathematical reasoning may help) how Y, R, and w_R develop over time, in the long run. Assume that the resource remains 'open access' throughout.

   ii. In Model 2 the resource price w_R is equal to its extraction cost. Explain why in a few words, and use this fact to find a simple expression for w_R.

   iii. Staying with Model 2, assume balanced growth and find expressions for $\dot{Y}/Y$ and $\dot{R}/R$.

   (b) Compare the models in their ability to (i) match and (ii) explain global aggregate observations of GDP growth, and growth rates of resource use and prices for resources such as metals and fossil fuels.

   (c) One of these models ignores resource scarcity altogether, whereas in the other the global economy 'falls off a cliff' due to scarcity.

   Discuss how we can build more realistic models of how the global economy may be affected by (and adapt to) resource scarcity or the need to reduce polluting emissions associated with resource use. Will it 'fall off a cliff'? Why, or why not?

(a) In Model 1, both Y and R will grow at rate g until the resource runs out, at which time the economy will collapse; w_r is zero throughout. In Model 2 there is no scarcity, so (assuming perfect markets) prices are equal to unit costs, and resource costs and equal to extraction costs. The extraction input is X, which is just final product, price 1. And each unit of X gives ϕ units of R, so $w_R = 1/\phi$. Show that $\dot{Y}/Y = \dot{R}/R = g$.

(b) Both models match the long-run trends reasonably well, although according to Model 1 the price of resources is zero rather than constant and positive. In Model 1 resources get sucked in when A_L grows, and there is no way to do without resources at all. This is not realistic. And the idea of open access resources free to extract is obviously wrong. Whereas in Model 2, if resources started to get scarce (which they won't given the assumptions, but will in reality) we would adapt by simply using less, without that having a terrible effect on production, as long as β is low. But how would this work in practice? Would be boost resource efficiency A_R? In the model, this scarcely helps. Would we switch between products? Or find other inputs? Model 2 doesn't have any answers to these questions, so doesn't explain much either.

(c) The main thing here is to discuss Solow's mechanisms, and (e.g.) models with directed technological change, or endogenous choice of consumption goods, or green technology shifts triggered by rising WTP for environmental quality. You could also discuss the more sophisticated extraction model but it's not really directly relevant given the phrasing of the question.

2. Assume an economy on an island with a single product, widgets. Widgets are made using labour and energy, in a Leontief production function:

$$Y = (A_Y L_Y)^{1-\alpha} E^{\alpha}.$$

Of production Y, X is used to make energy inputs and the rest is consumed. The flow of energy inputs E is as follows:

$$E = A_F L_F + A_R L_R,$$

where F denotes fossil fuels and R renewables. So energy may be produced using one or both of fossil and renewable sources, where the two are perfect substitutes. A_Y, A_F, and A_R are productivities, and L_Y, L_F and L_R are quantities of workers in each sector. All markets are perfect, and there is no scarcity. Normalize the price of a widget to 1 SEK.

Now assume that in addition to labour L there are researchers. A fixed number of researchers work on raising A_Y, and as a consequence $A_Y(t+1)/A_Y(t) = 1.2$ (one time period is 10 years). Furthermore, there is a fixed number of researchers Z in the energy sector, divided between Z_F and Z_R. And

$$A_F(t+1) = 0.95A_F(t) + \phi Z_F(t)[\sigma A_Y(t) + A_F(t)]$$
$$\text{and} \quad A_R(t+1) = 0.95A_R(t) + \phi Z_R(t)[\sigma A_Y(t) + A_R(t)].$$

Assume that $A_{Y0} = 100$, $A_{F0} = 10$, and $A_{R0} = 1$, while $Z = 5$, $\phi = 0.01$, and $\sigma = 0.4$. Finally, assume that researchers are allocated 'myopically' according to current factor shares.

(a) i. Find an expression for the price of energy w_E when it is generated from fossil fuels. Your expression should be in terms of A_F and w_L (the wage, which is the same for all workers). Find an equivalent expression when renewables are used.

ii. Explain why fossil fuels will be used exclusively, and why all researchers will be allocated to fossil research.

iii. Show that A_Y and A_F will grow at equal rates.

Note that when A_Y and A_F grow at equal rates we have balanced growth, and E, Y, and w_L also grow at the same rate, while w_E is constant.

The government discovers that fossil fuel burning is having severe negative effects on the quality of the environment, whereas renewables would have no such effects. Assume that a Pigovian tax (equal to marginal damages) would add $0.1w_L$ to the price of energy generated from fossil fuels.

(b) Find the market allocation if the Pigovian tax is applied at $t = 0$. In broad terms, how will the economy evolve? (Think about how the Pigovian tax changes over time.)

(c) Assuming that the society is patient (low social discount rate), this allocation will not be socially optimal. Explain why not, and discuss alternative (or additional) policies.

Discuss briefly what we can learn from the model regarding optimal regulation of CO_2 emissions from the burning of fossil fuels.

(a) $w_E = w_L/A_F$ or $w_E = w_L/A_R$, depending on the input used. Since $A_F > A_R$ and the inputs are perfect substitutes, we will use only fossil. And then since the fossil share is 100 percent, all the researchers will go here too. For the growth rates, divide the knowledge production function through by A_{Ft} to obtain $A_{Ft+1}/A_{Ft} = 0.95 + 0.01 \times 5 \times [0.4A_{Yt}/A_{Ft} + 1] = 1.2$.

(b) Without the tax, the price of fossil energy is w_L/A_F. With it, it is $w_L(1/A_F + 0.1)$. When $A_F = 10$ this doubles the fossil price, which isn't enough to make fossil more expensive than renewable. But as the economy grows, the tax will increase and the fossil price will increase. However, in the meantime A_R is falling. It is not obvious what will happen in the long run, but it will definitely take a very long time before renewables take over.

(c) The problem is that firms are so myopic. Renewables are fundamentally better than fossil (same underlying production costs, no damages) but because the initial technology is behind they never get used, because firms don't look to the future. (This could be because their patents run out after 10 years, for instance.) So in the model we would need a research subsidy to go along with the Pigovian tax. The subsidy would push researchers into the renewable sector, A_R would grow rapidly, and renewables would take over after a few periods.

In the real world firms aren't this myopic. But there is still a need for research subsidies for green technologies that are important but are also a long way from the market.

3. You are given the production and instantaneous utility functions in two models which provide alternative explanations of why consumers may shift towards more energy-intensive goods over time.

- Model 1.

  There are two products Y_1 and Y_2 produced by labour and energy respectively.

  $$Y_1 = A_L L;$$
  $$Y_2 = A_E E.$$

  Labour L is fixed, and energy is extracted at fixed unit cost. All markets are perfect. Instantaneous utility is a Cobb–Douglas function of the two:

  $$u = Y_1^{1-\alpha} Y_2^{\alpha}.$$

- Model 2.

  There is an infinite series of products Y_i, and the production function for product i is as follows:

  $$Y_i = (1/2^{i-1}) \min\{A_L L_{Yi}, A_E E_i/2^{i-1}\},$$

  where A is productivity, L_Y is labour in final-good production, E is the energy input, and A_E is fixed. Consumers have lexicographic preferences such that they always prefer to consume the good with the highest i that they can afford, given that they demand a minimum quantity.

In both models the productivities A_L and A_E each grow at the constant exogenous rate g, and the initial factor share of energy is approximately 5 percent.

(a) Consider Model 1. Show that the factor share of energy is constant, and explain what this implies about the growth rate of energy use given that the energy price is constant. What happens if energy efficiency A_E increases faster than A_L?

(b) Consider Model 2. Explain why, as A_L and A_E grow, consumers shift to more energy-intensive goods. What are the implications for the growth rate of energy use? What happens if energy efficiency A_E increases faster than A_L?

Swedes' spending on international flights rose rapidly between 1980 and 2018 (much more rapidly than GDP). The result was that energy use and carbon emissions from the sector grew rapidly, despite increasing efficiency of airplanes.

(c) Explain how each of the models above might be able to shed light on these observations, using the terms 'substitution effect' and 'income effect'. Which model do you think comes closest to the truth?

(a) The relative factor shares of energy and labour are given by $w_E E/(w_L L) = \alpha/(1-\alpha)$. So energy use will grow at the overall growth rate if the price is constant. This is true irrespective of growth rates of A_E and A_L.

(b) In Model 2 goods with higher i are more energy-intensive (the factor 2^{i-1} in the right-hand part of the production function), and also more expensive (the factor $1/2^{i-1}$ at the start). As productivity grows, consumers get richer. They always choose the good with the highest i they can afford, so the move 'up' to higher i and more energy intensity. Increases in A_E don't affect income much, but do affect energy intensity. So they cut energy use!

(c) Model 1, price falls, substitution effect. Model 2, income increases, income effect. Model 1 doesn't work because energy is only around 20 percent of the cost of flights. Model 2 is of course way to simple, but probable more relevant than Model 1 (in my opinion).

4. Consider the CES production function

$$X = [(A_1 X_1)^\epsilon + (A_2 X_2)^\epsilon]^{1/\epsilon},$$

where X_1 and X_2 are inputs and X is an output, while A_1 and A_2 are productivities. Markets are perfect.

(a) Derive an expression for $w_1 X_1/(w_2 X_2)$ in terms of the quantities X_1 and X_2, the productivities A_1 and A_2, and ϵ.

(b) Assume X_1 is labour and X_2 is a natural resource input, while Y is final product. Suggest an appropriate value for ϵ, and discuss—using theory and evidence—how changes in natural-resource supply may affect the factor share of resources in the short and the long run.

(c) Assume X_1 and X_2 are alternative primary energy inputs (e.g. oil and renewables) and X is an intermediate input into the final good production function $Y = (A_L L)^{1-\alpha} X^\alpha$. Suggest an appropriate value for ϵ, and discuss—using theory and evidence—how changes in supply of one of the resources may affect its factor share in the short and the long run.

(a) You should get

$$\frac{w_1 X_1}{w_2 X_2} = \left(\frac{A_1 X_1}{A_2 X_2}\right)^\epsilon.$$

(b) Your choice of ϵ should be negative. And in the short run an increase in resource supply will push the share of the resource down, because the price will fall steeply. But in the long run the lower price will lead to more use of the resource (and maybe less resource-saving technological change) and the share will go up again. Give evidence from e.g. how oil prices change over time, with short-run volatility and long-run factor share rather stable.

(c) Now you should choose positive ϵ. Now an increase in supply won't affect the price much, and the share will go up. If DTC effects are strong, the share may keep on increasing due to more investment in knowledge augmenting that input. But we don't tend to see this in reality. Give evidence from long-run trends in e.g. iron and aluminium factor shares.