Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen and paper.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. Assume an economy described by the following equations:

$$Y_t = \min\{A_{Lt}L_t, A_{Rt}R_t\}; \tag{1}$$
$$A_{Lt+1} = (1+\theta)A_{Lt}; \tag{2}$$
$$\sum_{t=0}^{\infty} R_t \leq S_0. \tag{3}$$

Furthermore, we have that $A_{L0}L_0 \ll A_{R0}S$. Labour L is fixed, and hired on perfect markets, whereas the resource R is free to extract and of 'open access' character, i.e. no individual or group has property rights over the resource.

   (a) Interpret these equations.

   (b) Now assume that periods are 30 years long, in period 0 we have $A_{L0} = A_{R0} = 1$, $L = 1$, $S_0 = 10$, and $\theta = 1$. Characterize the development of this economy in the following two cases:

      i. A_R is constant;

      ii. A_R increases by 1 every period.

   (c) Discuss the extent to which this model—or adaptations of it—can help us to explain historical observations and predict the future.

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2. Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour L and trees R, with factor-augmenting technology levels A_L and A_R. It can be written

$$Y = [(A_L L)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}.$$

The parameter $\epsilon = -1$. There are 11 people on the island, of whom 10 work in production and 1 in research, and 10 trees/year wash up on the shore. All markets are perfect.

   (a) Assume that—in year t—the islanders have a technology called 'penknives' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $A_L = 1$, $A_R = 9$. Calculate GDP per capita, and the relative prices and factor shares of labour and trees.

   (b) Assume that the islanders' knowledge production functions are as follows, where z_l and z_r are the proportions of the year that the single researcher spends working on labour-augmenting and tree-augmenting knowledge:

$$A_{Lt+1} = A_{Lt}(1 + 0.1z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(1 + 0.9z_{rt+1}).$$

   Furthermore, assume that relative investments z_l/z_r in period $t+1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t.

i. What are GDP per capita, relative prices, and relative factor shares in year $t+1$, if the flow of trees remains constant?

ii. What happens in the long run if the flow of trees diminishes towards zero over time?

iii. Discuss very briefly the interpretation of rises in A_L and rises in A_R.

(c) Discuss the extent to which the model economy is a good metaphor for the real global economy, and therefore the likely similarities between what happens in the model economy and what may happen in the global economy if resource and energy flows must be reduced.

3. Assume an economy with competitive markets in which total aggregate production is a function of labour-intensive and energy-intensive production, as follows:

$$Y = Y_1^{\alpha} Y_2^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_1 = a_l L,$$

where a_l is labour-augmenting knowledge and L is labour, which is fixed. The energy-intensive good is produced according to the following production function

$$Y_2 = a_r R,$$

where a_r is energy-augmenting knowledge, and R is the energy flow. Energy is extracted using the final product as an input, and one unit of final product yields one unit of energy. Hence if we normalize the price of the final product to 1, the energy price is also 1.

(a) i. Find the relative shares in total product of Y_1 and Y_2. (That is, find $p_1 Y_1/(p_2 Y_2)$, where p_1 and p_2 are the prices of the two goods Y_1 and Y_2.)

ii. Find total energy use R for a given state of the economy. (This is, when L, a_l, and a_r are all fixed and known.)

iii. Assume that a regulator wants to reduce R, and that she can either boost w_r through a tax, or a_r through a research subsidy. Explain which option she should choose in this economy.

(b) Discuss to what extent the above model is relevant to real economies in which the energy share of the most energy-intensive products is typically only about 15 or 20 percent, rather than 100 percent as in the model.

4. Hotelling's analysis with zero extraction costs shows that the real price of non-renewable resources should rise at the interest rate. However, this is not consistent with historical data.

   (a) How can we explain historical data regarding non-renewable resource prices, and predict future prices?

   (b) Discuss likely trends in the very long run regarding the prices of energy, land, and non-renewable resources such as metal ores (iron, copper, rare earths, etc.). Explain your reasoning!