Department of Economics
Rob Hart

January 2015

Examination

Sustainable Development, NA0167.

Rules

Permitted aids: Pen and paper.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Resource prices and quantities in neoclassical theory.
3. Directed technological change and sustainability.
4. Consumption, rebound, and sustainability.

1. Consider the neoclassical growth model with zero population growth, and add the need for a resource input in the production function. The flow of the resource input is denoted R, there is a total stock of the resource S, and there is no substitute for the resource. There is exogenous labour-augmenting technological progress at a constant rate. The discount rate (interest rate) is exogenously fixed at ρ.

$$\begin{aligned}
Y &= (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta}, \\
\dot{A}/A &= g_A, \\
\dot{K} &= sY - \delta K, \\
S &\geq \int_0^{\infty} R_t \mathrm{d}t.
\end{aligned}$$

(a) Assume that there exists a balanced growth path (b.g.p.) on which all variables grow at constant rates. Find the growth rate of Y on this b.g.p. in terms of parameters and $\dot{R}/R$.

(b) Consider a competitive final-good producer who makes profits $\pi = (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta} - w_l L - w_k K - w_r R$. (The price of Y is normalized to 1.) Take the first-order condition in R to find an expression for w_r in terms of Y and R.

(c) Use your previous answers to find an expression for the growth rate of w_r in terms of parameters and $\dot{R}/R$.

(d) Imagine that you own a small fraction of the total resource stock S, and that the resource is traded on a perfect market (there are many other small suppliers). How do you decide when to sell your resource stock? Given that the other suppliers reason in the same way as you, what must be the growth rate of w_r? Now use your answer to part (c) to find the growth rate of R.

(e) Discuss briefly the extent to which this model might give a useful —although highly simplified—picture of economic growth and resource use in the long run.

---

2. Consider an economy with a constant population and a constant interest rate r, and the following aggregate production function:

$$y = (a l_y)^{1-\alpha} x^{\alpha}.$$

Here α is a parameter between 0 and 1, a is labour productivity, and l_y and x are the quantities of labour and resources used in production. The resource flow x is given by the following extraction function:

$$x = l_x a/b.$$

Labour inputs in extraction are l_x, the productivity of that labour is a (as above), and b is an inverse productivity factor representing the difficulty of extracting the resource: the *depth* of the marginal resource.

The productivity index a grows exogenously, and total labour L is fixed:

$$\dot{a}/a = \theta_a$$
$$L = l_x + l_y.$$

Finally, we assume that all markets are perfect, and we have a unit continuum of resource owners each with identical inhomogeneous endowments.

(a) Find an expression for the resource price p in terms of y and x, by taking the first-order condition on a final-good producer's profit function.

(b) Assume a b.g.p. on which quantities of labour are constant and depth b grows at a constant rate. Find expressions for the growth rate of x, p, and y on such a b.g.p.

(c) Assume a primitive economy in which resource extraction is just beginning. What can we say about b? Characterize the b.g.p.! What happens over time?

(d) Assume a 'mature' b.g.p. on which b grows at a constant strictly positive rate. For concreteness assume that $\dot{b}/b = \theta_a$ on this path. Characterize the b.g.p.!

(e) Finally, characterize the development of the economy if the resource is close to exhaustion, and there are no substitutes.

---

3. [A]s the earth's supply of particular natural resources nears exhaustion, and as natural resources become more and more valuable, the motive to economize those natural resources should become as strong as the motive to economize labor. The productivity of resources should rise faster than now—it is hard to imagine otherwise.
[Solow, *Is the end of the world at hand?*, Challenge, 1973, p47.]

(a) Over the last 300 years the price of energy has been falling compared to the price of labour.

   i. Explain why, in theory, this might lead labour-augmenting knowledge to grow faster than energy-augmenting knowledge.

   ii. Discuss evidence.

(b) Over the next 50 years the price of energy may well rise relative to the price of labour. Will this lead to rapid increases in the efficiency of energy use in sectors such as lighting and transport? Discuss theory and evidence.

---

4. Assume an economy with competitive markets in which total aggregate production is a function of labour-intensive and energy-intensive production, as follows:

$$Y = Y_1^{\alpha} Y_2^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_1 = a_l L,$$

where a_l is labour-augmenting knowledge and L is labour, which is fixed. The energy-intensive good is produced according to the following production function

$$Y_2 = a_r R,$$

where a_r is energy-augmenting knowledge, and R is the energy flow. The price of energy, w_r, is fixed in relation to the wage w_l:

$$w_r = \psi w_l,$$

where ψ is a positive parameter. Any amount of energy R can be supplied at this price.

(a) i. Find the relative shares in total product of Y_1 and Y_2. (That is, find $p_1 Y_1/(p_2 Y_2)$, where p_1 and p_2 are the prices of the two goods Y_1 and Y_2.)

ii. Consider unit production costs in order to find p_1 and p_2 in terms of w_l, ψ, a_l, and a_r.

iii. Find total energy use R for a given state of the economy. (This is, when L, a_l, a_r, and ψ are all fixed and known.)

iv. Assume that a regulator wants to reduce R, and that she can either boost ψ (and hence w_r) through a tax, or a_r through a research subsidy. Explain which option she should choose in this economy.

(b) Discuss to what extent the above model is relevant to real economies in which the energy share of the most energy-intensive products is typically only about 15 or 20 percent, rather than 100 percent as in the model.