Department of Economics
Rob Hart

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Examination

Sustainable Development, NA0115.

Rules

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Resource prices and quantities in neoclassical theory.
3. Directed technological change and sustainability.
4. Consumption, rebound, and sustainability.

1. Assume an economy described by the following equations:

$$Y = \min\{A_K K, A_R R\}; \tag{1}$$

$$K_{t+1} - K_t = sY_t - \delta K_t; \tag{2}$$

$$S_0 \geq \sum_{t=0}^{\infty} R_t. \tag{3}$$

Furthermore, we have that $A_K K_0 < A_R S_0$. Capital K is traded on perfect markets, whereas the resource R is free to extract and of 'open access' character, i.e. no individual or group has property rights over the resource.

   (a) Interpret these equations.

   (b) Now assume that periods are 30 years long, in period 0 we have $A_{K0} = A_{R0} = 10$, $K_0 = 1$, $S_0 = 10$, and (in all periods) $s = 0.11$ and $\delta = 0.1$. Characterize the development of this economy in the following two cases.

      i. A_K and A_R are constant.

      ii. A_K is constant whereas A_R increases by 10 every period.

   (c) Discuss the extent to which this model—or adaptations of it—can help us to explain historical observations and predict the future.

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2. (a) Assume you own a quantity of a resource S, which you can extract at zero cost at any time. Furthermore, you know with certainty both the demand function for the resource,

$$p_t = A + Bq_t^{-\epsilon},$$

   and the interest rate r. You have a monopoly over supply. Note that A is a parameter which may be positive, negative, or zero, B and ϵ are positive parameters, r is constant, and p_t and q_t are the price and quantity of the resource in period t.

      i. Set up your constrained profit-maximization problem as a Lagrangian, and solve it first for p_t, then for p_{t+1}/p_t.

      ii. What can we say about the rate of price increase for a monopolist compared to the perfect market? What does this imply about the initial price, and the longevity of the resource?

   (b) In practice we rarely see sustained increases in resource prices; indeed, they seem to be remarkably constant in the long run. Why might this be? Discuss and evaluate at least two alternative explanations. What are the policy implications?

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3. [A]s the earth's supply of particular natural resources nears exhaustion, and as natural resources become more and more valuable, the motive to economize those natural resources should become as strong as the motive to economize labor. The productivity of resources should rise faster than now—it is hard to imagine otherwise.

[Solow, *Is the end of the world at hand?*, Challenge, 1973, p47.]

(a) Over the last 300 years the price of energy has been falling compared to the price of labour. Has this led to a failure of energy-augmenting knowledge to grow as fast as labour-augmenting knowledge? Discuss evidence.

(b) Over the next 50 years the price of energy may well rise relative to the price of labour. Will this lead to rapid increases in the efficiency of energy use in sectors such as lighting and transport? Discuss theory and evidence.

Turn over

4. Assume an economy in which total aggregate production is a function of labour-intensive and resource-intensive production, as follows:

$$Y = Y_L^{\alpha} Y_R^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_L = k_l L,$$

where k_l is labour-augmenting knowledge and L is labour. The resource-intensive good is produced according to the following production function, where C is a clean input and D is a dirty input, and ϵ is a parameter less than one:

$$Y_R = [(k_c C)^{\epsilon} + (k_d D)^{\epsilon}]^{1/\epsilon}.$$

Finally, knowledge stocks grow together, exogenously:

$$k_l = k_c = k_d;$$
$$\dot{k}_l / k_l = \theta.$$

(a) i. Find the shares in total product of Y_L and Y_R.

ii. Find the shares in total product of L and R, where $R = C + D$. (That is, find $w_l L$ and $w_c C + w_d D$.)

iii. Derive separate expressions for the relative factor shares of C and D (i.e. $w_c C/(w_d D)$), first in terms of quantities, then in terms of prices.

iv. Assume that initially only factor D is available, but that at some time T factor C also appears on the market. What happens?

(b) Discuss the extent to which this model—or adaptations of it—can help us to design policy to reduce carbon dioxide emissions from the burning of fossil fuels.