Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. (a) Compare the following two models in (i) their ability to explain historical data about resource extraction rates and resource prices, and (ii) their ability to help us predict the future effects of resource scarcity.

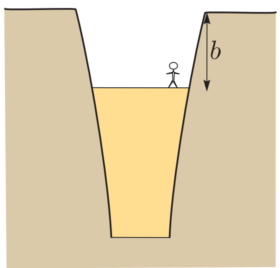
   - Model 1 (the standard DHSS model with competitive markets):

$$\begin{aligned} Y &= (AL)^{1-\alpha-\beta} K^\alpha R^\beta, \\ \dot{A}/A &= g, \\ \dot{K} &= sY - \delta K, \\ C &= (1-s)Y, \\ S &\geq \int_0^\infty R_t \mathrm{d}t. \end{aligned}$$

   - Model 2 (the DHSS model with a resource in infinite supply but costly to extract, and competitive markets):

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta; \\ \dot{A}_L/A_L &= g; \\ \dot{K} &= s(Y - X) - \delta K; \\ C &= (1-s)(Y-X) \\ R &= \phi X. \end{aligned}$$

   (b) Discuss an extension to the latter model in which we instead assume that the resource stock is finite and inhomogeneous. More specifically, assume many (competitive) resource owners who each own a stock which looks something like that in the picture below, where b indicates the depth of the stock, and unit extraction costs increase in b. Can this model help us to both explain and predict resource trends?



2. Assume an economy on an island with a single product, widgets. Widgets are made using labour plus either coal or gas, quantities C and D (in tons/year), which are then converted into energy E (in KJ/year):

$$E = A_C C + A_D D.$$

The production function for widgets is Leontief,

$$Y = \min\{A_L L, E\}.$$

Coal and gas are extracted using final goods (widgets) with fixed productivity, set to 1:

$$C = X_C \quad \text{and} \quad D = X_D.$$

A_L, A_C, and A_D are productivities, X_C and X_D are quantities of widgets sent to the energy sector, and L is labour. All markets are perfect, and there is no scarcity. Normalizing the price of widgets to 1 SEK, the prices of coal and gas are thus 1 SEK/ton.

(a) i. Find the condition for coal to be used rather than gas.
ii. Assume that coal is used exclusively, and find an expression for expenditure on coal as a fraction of the value of production Y.
iii. How much is spent on labour inputs?

Now assume that in addition to labour L there is a fixed number of researchers Z, who can be assigned to raising the productivity levels in the economy. For each productivity A_i, the effect of research effort is as follows:

$$A_i(t+1) = A_i(t)(0.999 + \phi Z_i(t)).$$

Furthermore, assume that $A_L = 1$, $A_C = 20$, and $A_D = 5$, while $Z = 20$, $\phi = 0.001$. Finally, assume that researchers are allocated 'myopically' according to current factor shares.

(b) What is the market allocation of researchers between Z_L, Z_C, and Z_D? How will the economy evolve (growth rates of GDP and resource use, factor shares of labour and the resources)?

The government discovers that coal burning is having severe negative effects on the quality of the environment, whereas gas would have no such effects. A pigovian tax (equal to marginal damages) would be 1 SEK/ton.

(c) Find the market allocation if the pigovian tax is applied. In broad terms, how will the economy evolve?

(d) Assuming that the society is patient (low social discount rate), this allocation will not be socially optimal. Explain why not, and discuss alternative (or additional) policies. Discuss what, if anything, we can learn from the model regarding optimal regulation of CO_2 emissions from the burning of fossil fuels.

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3. Discuss the following statement.

> Changing consumption patterns are the cause of the rapid growth of global primary energy use illustrated in Figure 1. This implies that rebound effects are very powerful, hence increases in energy efficiency will not on their own reduce energy consumption.

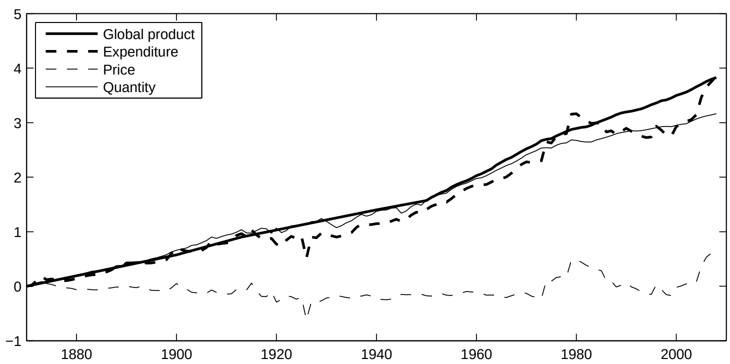

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[1]

[1] Energy: Coal, oil, natural gas, and biofuel.

4. Assume an economy controlled by a social planner with a single final good produced in quantity Y using inputs of labour L and electricity E. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} E^{\alpha}(1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the resource has a small factor share). A_L and L grow exogenously at constant rates. Electricity E is produced using coal X_1, and we choose units such that

$$E = X_1,$$

i.e. the flow of energy is equal to the flow of coal. The extraction cost of coal, w_1, is constant. Furthermore, burning a unit of coal leads to ϕ units of polluting emissions,

$$D = \phi X_1.$$

Utility U is production Y minus total extraction costs, $w_1 X_1$, so

$$U = (A_L L)^{1-\alpha} E^{\alpha}(1 - \psi D) - w_1 X_1.$$

(a) i. Write down an expression for utility in terms of X_1, and find an expression for $\partial U / \partial X_1$.

ii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very small. (Hint: What does this imply about pollution damages per unit of X_1, compared to extraction costs?)

iii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very large.

iv. Describe the development of the economy over time assuming that at $t = 0$, $A_L L$ is very small.

(b) Explain carefully what difference it would make if there were an alternative method of producing electricity using an input X_2 that was more expensive ($w_2 > w_1$) but emissions-free.

(c) Discuss the relevance of the model to explaining and predicting real world phenomena.