

# Brief suggested answers

EGSD Examination, February 2017.

*Note that longer answers may be required for full marks. For instance, it is important to show your working in calculation questions. And for discussion or essay questions my answers are intended as an outline.*

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1. (a) i. Consider the representative resource owner, who has two assets, the resource stock in the ground, and money in the bank. Assume for simplicity that she starts with zero money in the bank. Then the value of her portfolio is  $w_R S$ . Now assume that  $w_R$  is growing at rate  $g$ . Then if she leaves the resource in the ground, her portfolio will increase in value at rate  $g$ . And if she extracts the whole resource stock and puts the money ( $w_R S$ ) in the bank, then the value of her portfolio will increase at rate  $\rho$ .

If  $\rho = g$  then she is indifferent between extracting and not extracting. This is OK. But if  $g > \rho$  then she will extract nothing, and nor will any of the other resource owners. This cannot be an equilibrium. And if  $\rho > g$  then she will extract everything, as will all of the other resource owners. This cannot be an equilibrium either. So the only reasonable possibility is that  $\rho = g$ , the resource price grows at the interest rate:

$$\frac{\dot{w}_R}{w_R} = \rho.$$

- ii. The representative final-good producer's profit function is:

$$\pi = (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta - (w_L L + w_K K + w_R R).$$

From the FOC in  $R$  we obtain  $w_R R = \beta Y$ , so the resource captures a proportion  $\beta$  of total returns. And (differentiating w.r.t. time) we have

$$\begin{aligned} \frac{\dot{w}_R}{w_R} + \frac{\dot{R}}{R} &= \frac{\dot{Y}}{Y}, \\ \text{hence} \quad \frac{\dot{R}}{R} &= \frac{\dot{Y}}{Y} - \rho. \end{aligned}$$

- (b) i. Given the extraction function  $R = \phi X$ , where  $X$  is final goods and  $\phi$  is a constant, the price of the resource is constant.
- ii. From above, but with constant price, we now have

$$\frac{\dot{R}}{R} = \frac{\dot{Y}}{Y}.$$

- (c) The historical data shows no long-run trend in resource prices, and extraction rate tending to track growth in GDP. This is broadly in line with the second model, but completely at odds with the first model.

However, for predicting the future the first model is not much use, since it's obvious that we can't continue indefinitely with an exponential increase in resource extraction rates. Maybe something like the first model will apply in the very long run?

This is the key, but for full points you should go deeper, e.g. by briefly mentioning the extended model from the January 2017 exam question.

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2. (a) GDP = 5 houses/year. By symmetry, the factor shares are equal (50 percent each).
- (b) i. One researcher works in one sector, the other in the other sector. So both  $A_L$  and  $A_R$  rise by 2 percent, GDP rises by 2 percent, and the factor shares remain equal.
- ii. Now we need calculus. Since relative investments are based on shares in previous period, they are equal, and  $A_L$  and  $A_R$  remain equal. But from the first-order conditions we have that

$$\frac{w_L L}{w_R R} = \left( \frac{A_L L}{A_R R} \right)^\epsilon,$$

and the relative factor shares are 1 : 10, i.e. trees take ten times the share of labour. And GDP drops to just 0.95 houses/year.

- iii. Given the high share of trees, the researchers switch to working almost exclusively on tree-augmenting knowledge, which rises rapidly compared to labour-augmenting knowledge, bringing the shares back towards parity. In the long run we have  $A_L/A_R = 20$ , and equal shares again.
- (c) This is about Solow's three mechanisms: DTC economizing on the input (above), DTC favouring a substitute input, and shifts in consumption patterns. Typically I would argue that shifts to substitutes, and shifts in consumption patterns, are more important than economization. But the main thing is to describe the mechanisms and give examples.

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3. This question should normally be tackled in three stages: (i) What is the cause of the rapid growth? (ii) Does this imply that rebound effects are very powerful? (iii) What are the implications for the benefits of energy efficiency. Here I give some hints about what you could take up.

Broadly, it seems to be true that the rapid growth in energy consumption—tracking global product up to 1974—is indeed largely due to

changing consumption patterns. An important alternative explanation would be a lack of energy-augmenting technological progress, but we know that such progress has in fact been rapid.

It is not true that this implies very powerful rebound, since the changes could be caused either by substitution effects (which are strongly linked to rebound) or income effects (which are not).

To the extent that income effects are the cause (and it seems likely that they are an important cause) increases in energy efficiency do help to keep energy use down; without the historical improvements we have observed, the global increases in energy consumption would have been even larger.

4. (a) The marginal costs are input costs plus damages:

$$MC_1 = w_1 + \psi(A_L L)^{1-\alpha} R^\alpha;$$

$$MC_2 = w_1(1 + \gamma).$$

The marginal benefits are identical for the two inputs:

$$MB_1 = MB_2 = \alpha Y/R.$$

They are perfect substitutes.

- (b) i. The condition is

$$w_1 \gamma / \psi = (A_L L)^{1-\alpha} R^\alpha,$$

and it implies that as  $A_L$  and  $L$  grow (also causing  $X_1$  and hence  $R$  to grow) there comes a point when it is better to use input  $X_2$  instead of  $X_1$ .

- ii. If the economy is optimally regulated then  $R$  will initially be produced using  $X_1$ , and  $X_1$  (and also pollution flows) will grow at a high rate (close to the overall growth rate). As GDP increases, pollution damages become significant, and brake the growth in  $X_1$  somewhat (this could be through the use of a Pigovian tax, for instance). Then, when the condition above is fulfilled, the tax becomes so high that the economy switches totally to input  $X_2$ . (Alternatively, input  $X_1$  is banned.)
- (c) The model is relevant to the EKC, i.e. the environmental Kuznets curve hypothesis. The basic observation behind the EKC is that—for many pollutants, in many countries—pollution flows tend to first rise, and then fall. This is exactly what is predicted by the model.

*Having stated this, you could develop your answer in 1000 different ways. For instance, you could discuss a particular case which supports the model (lead, asbestos, etc.), or you could discuss whether the model sheds light on the problem of carbon dioxide emissions, even though these are still rising in most countries, and the global aggregate is still rising steeply.*