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Examination

Sustainable Development, NA0167.

Rules

Permitted aids: Pen and paper.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Resource prices and quantities in neoclassical theory.
3. Directed technological change and sustainability.
4. Consumption, rebound, and sustainability.

1. Consider a simple DHSS model with zero population growth. Labour, capital, and a non-renewable resource are essential for production. The flow of the resource input is denoted R, there is a total stock of the resource S, and there is no substitute for the resource. The discount rate (interest rate) is exogenously fixed at ρ. All markets are competitive. The equations of the model are as follows.

$$\begin{aligned} Y &= (AL)^{1-\alpha-\beta} K^\alpha R^\beta, \\ \dot{A}/A &= g_A, \\ \dot{K} &= sY - \delta K, \\ S &\geq \int_0^\infty R_t \mathrm{d}t. \end{aligned}$$

Assume that $g_A = 0.02$ while $\delta = 0.1$.

(a) Describe how the economy develops over time (you may use mathematics but it is not essential).

(b) What lessons can we learn from the model about sustainable management of non-renewable resources in reality?

Now assume that $g_A = 0$ and $\delta = 0$, and that the goal is to maintain constant production forever, i.e. to ensure that $\dot{Y}/Y = 0$.

(c) Describe in general terms how the economy must develop over time if this goal is to be met (you do not need to use mathematics). What lessons can we learn from this model about sustainable management of non-renewable resources in reality?

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2. Consider an economy with a constant population and a constant interest rate r, and the following aggregate production function:

$$y = (al_y)^{1-\alpha} x^\alpha.$$

Here α is a parameter between 0 and 1, a is labour productivity, and l_y and x are the quantities of labour and resources used in production. The resource flow x is given by the following extraction function:

$$x = l_x a/b.$$

Labour inputs in extraction are l_x, the productivity of that labour is a (as above), and b is an inverse productivity factor representing the difficulty of extracting the resource: the *depth* of the marginal resource. Labour productivity a grows exogenously, and total labour L is fixed:

$$\begin{aligned} \dot{a}/a &= \theta_a \\ L &= l_x + l_y. \end{aligned}$$

All markets are perfect, and there is a unit continuum of resource owners each with identical inhomogeneous endowments.

(a) Find an expression for the resource price p in terms of y and x, by taking the first-order condition on a final-good producer's profit function.

(b) Assume a b.g.p. on which quantities of labour are constant and depth b grows at a constant rate. Find expressions for the growth rates of x, p, and y on such a b.g.p.

(c) Assume a primitive economy in which resource extraction is just beginning. What can we say about b? Characterize the b.g.p.! What happens over time?

(d) Assume a 'mature' b.g.p. on which b grows at a constant strictly positive rate. For concreteness assume that $\dot{b}/b = \theta_a$ on this path. Characterize the b.g.p.!

(e) Finally, characterize the development of the economy if the resource is close to exhaustion, and there are no substitutes.

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3. Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour A and trees B, quantities Q_a and Q_b with factor-augmenting technology levels K_a and K_b. It can be written
$$Y = [(K_a Q_a)^\epsilon + (K_b Q_b)^\epsilon]^{1/\epsilon}.$$
The parameter $\epsilon = -1$. There are 10 people on the island who all work in production, and 10 trees/week wash up on the shore. All markets are perfect. The price of houses is normalized to 1.

(a) i. Assume that the islanders have a technology called 'penknives' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $K_a = 0.1$, $K_b = 1$. What is the GDP per capita on the island?

ii. Now assume that the islanders obtain a technology called 'sawmills', corresponding to $K_a = 10$, $K_b = 1$. What is GDP per capita now?

(b) Calculate the *prices* and *relative factor shares* of labour and trees in (a) and (b) above.

(c) Assume that the islanders' knowledge production functions are as follows, where z_a and z_b are investments, and ζ_a, ζ_b, and ϕ are positive parameters:
$$K_{at+1} = K_{at} z_{at+1}^\phi / \zeta_a;$$
$$K_{bt+1} = K_{bt} z_{bt+1}^\phi / \zeta_b.$$
Furthermore, assume that relative investments z_a/z_b are equal to relative factor shares $p_a Q_a/(p_b Q_b)$. What happens in the long run if the flow of trees diminishes towards zero over time?

(d) Discuss the relevance of the model for understanding how the global economy might adapt to diminishing resource availability.

4. Assume an economy with competitive markets in which total aggregate production is a function of labour-intensive and energy-intensive production, as follows:

$$Y = Y_1^{\alpha} Y_2^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_1 = a_l L,$$

where a_l is labour-augmenting knowledge and L is labour, which is fixed. The energy-intensive good is produced according to the following production function

$$Y_2 = a_r R,$$

where a_r is energy-augmenting knowledge, and R is the energy flow. The price of energy, w_r, is fixed in relation to the wage w_l:

$$w_r = \psi w_l,$$

where ψ is a positive parameter. Any amount of energy R can be supplied at this price.

(a) i. Find the relative shares in total product of Y_1 and Y_2. (That is, find $p_1 Y_1/(p_2 Y_2)$, where p_1 and p_2 are the prices of the two goods Y_1 and Y_2.)

ii. Find p_1 and p_2 in terms of w_l, ψ, a_l, and a_r.

iii. Find total energy use R for a given state of the economy. (This is, when L, a_l, a_r, and ψ are all fixed and known.)

iv. Assume that a regulator wants to reduce R, and that she can either boost ψ (and hence w_r) through a tax, or a_r through a research subsidy. Explain which option she should choose in this economy.

(b) Discuss to what extent the above model is relevant to real economies in which the energy share of the most energy-intensive products is typically only about 15 or 20 percent, rather than 100 percent as in the model.