Department of Economics
Rob Hart

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Example examination

Sustainable Development, NA0167.

Rules

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) There is one question on each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. Assume an economy the behaviour of which is described by the following aggregate functions.

$$\begin{aligned} Y &= AK^{\alpha}Q^{\beta}L^{1-\alpha-\beta} \\ \dot{A}/A &= g_A \\ \dot{L}/L &= n \\ \dot{K} &= sY - \delta K. \end{aligned}$$

Here Y is aggregate production of the single final product, A is productivity, K is capital (foregone consumption of the final product), Q is land, which is fixed, L is labour, and α, β, g_A, n, s, and δ are all parameters.

(a) Interpret these equations.

(b) Now rewrite the production function to obtain an expression for Y in terms of K/Y, and solve for g_Y and g_y on a balanced growth path. (Note that y is production per capita, and g indicates a growth rate.)

(c) Interpret your results.

(d) The model omits energy and mineral inputs. Could it nevertheless be a useful baseline for a model of the very long run? Explain.

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2. Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour A and trees B, quantities Q_a and Q_b with factor-augmenting technology levels K_a and K_b. It can be written

$$Y = [\alpha(K_aQ_a)^{\rho} + \beta(K_bQ_b)^{\rho}]^{1/\rho}.$$

Parameters α and β are both equal to 1, whereas $\rho = -1$. There are 10 people on the island who all work in production, and 10 trees/week wash up on the shore. All markets are perfect. The price of houses is normalized to 1.

(a) i. Assume that the islanders have a technology called 'penknives' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $K_a = 0.1$, $K_b = 1$. What is the GDP per capita on the island?

ii. Now assume that the islanders obtain a technology called 'sawmills', corresponding to $K_a = 10$, $K_b = 1$. What is GDP per capita now?

(b) Calculate the *prices* and *relative factor shares* of labour and trees in (a) and (b) above.

(c) Assume that the islanders' knowledge production functions are as follows, where z_a and z_b are investments, and ζ_a, ζ_b, and ϕ are positive parameters:

$$K_{at+1} = K_{at} z_a^{\phi} / \zeta_a;$$
$$K_{bt+1} = K_{bt} z_b^{\phi} / \zeta_b.$$

Furthermore, assume that relative investments z_a/z_b are equal to relative factor shares $p_a Q_a/(p_b Q_b)$. What happens in the long run if the flow of trees diminishes towards zero over time?

(d) Discuss the relevance of the model for understanding how the global economy might adapt to diminishing resource availability.

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3. "To the extent that it is impossible to design around or find substitutes for expensive natural resources, the prices of commodities that contain a lot of them will rise relative to the prices of other goods and services that don't use up a lot of resources. Consumers will be driven to buy fewer resource-intensive good and more of other things."[1]

   Discuss the extent to which this flexibility on the part of consumers may make the transition to a climate-friendly global economy harder to achieve, rather than easier.

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4. Assume you own a quantity of a resource Q, which you can extract at zero cost at any time. Furthermore, you know with certainty both the price path of the resource on the market, and the interest rate.

   (a) i. Explain intuitively your decision rule for when to extract and sell the resource.
   ii. Derive the rule mathematically by setting up a Lagrangian function.
   iii. You have found the rule for the rate of price increase. How is the *level* of the market price determined at a given time?

   (b) In practice we rarely see sustained increases in resource prices; indeed, they seem to be remarkably constant in the long run. Why might this be? Discuss and evaluate at least two alternative explanations. What are the policy implications?

[1] Robert Solow, Challenge, 1973, p.47.