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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. (a) Consider the DHSS model with competitive markets, zero extraction costs, a fixed saving rate, and constant population:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta, \\ \dot{A}_L / A_L &= g, \\ \dot{K} &= sY - \delta K, \\ C &= (1-s)Y, \\ S &\geq \int_0^\infty R_t \mathrm{d}t. \end{aligned}$$

The market interest rate is constant, denoted ρ.

i. Explain intuitively how the resource price grows in this model economy, based on supply considerations.

ii. Set up the representative final-good producer's problem, take the first-order condition in R, and use the result (plus part (i) above) to find an equation linking $\dot{R}/R$ to $\dot{Y}/Y$ and ρ.

(b) The above results do not match observations for non-renewable natural resource prices and quantities over the last 200 years.

Suggest alternative assumptions about the resource stock and extraction costs which, when integrated into the above model, yield results which better match the data. Specify your assumptions as exactly as you can, e.g. with the help of a flow diagram of the economy, or equations, or both.

(c) Suggest a further adaptation to the model which would allow us to investigate the potential to substitute between alternative resource inputs in production, and discuss its relevance to a problem of your choice, either involving resource scarcity or pollution.

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2. Assume an economy on an island with a single product, hammers. The production function is CES, with inputs of labour L_Y and coal R, with factor-augmenting technology levels A_L and A_R.[1] It can be written

$$Y = [(A_L L_Y)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}.$$

The price of hiring labour is normalized to A_L (so $w_L = A_L$). Meanwhile, coal is extracted from infinite homogeneous stocks by firms with the extraction function[2]

$$R = \phi A_L L_R.$$

All markets are perfect. Since total costs in the coal sector are $w_L L_R = A_L L_R$, the price of coal w_R is constant: $w_R = 1/\phi$.

(a) i. Set up the representative hammer-producer's static profit-maximization problem, and use it to derive an expression for the relative factor shares of labour and coal in terms of A_L, A_R, L_Y, and R.

[1] Iron is also needed, but it is very cheap and hence ignored in the production function. Units: hammers wkr^{-1} year^{-1} and hammers ton-of-coal^{-1}.

[2] So total labour L is divided between production and extraction. Later on we add research labour too.

ii. Multiply each side of this expression by $(w_L L/(w_R R))^{-\epsilon}$, and rearrange, in order to obtain an expression for the relative factor shares of labour and coal in terms of A_L, A_R, w_L, and w_R.

iii. Recall that $w_R = 1/\phi$, and $w_L = A_L$. Substitute in to your answer to part (ii) above, to obtain an expression for the relative factor shares of labour and coal in terms of A_R and parameters ϕ and ϵ.

(b) i. Assume that $\phi = 361$ and $\epsilon = -1$, while $A_L = 1$, $A_R = 1$. Find the factor shares of labour and coal in the final good sector, in percent. That is, find $(w_L L_Y/Y) \times 100$ and $(w_R R/Y) \times 100$.

ii. Assume that $L = L_Y + L_R = 1000$, and recall that the wage in the extraction sector is the same as the wage in the final-good sector. What is the allocation of labour between the two sectors?

(c) Assume that an additional 20 islanders work in the research sector, and that the islanders' knowledge production functions are as follows, where z_l and z_r are the numbers of researchers working on labour-augmenting and iron-augmenting knowledge respectively:

$$A_{Lt+1} = A_{Lt}(0.999 + 0.001 z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(0.999 + 0.001 z_{rt+1}).$$

Furthermore, assume that relative investments z_l/z_r in period $t+1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t. (So you do not need to set up and solve the full dynamic problem.)

i. Show that the economy is on a balanced growth path on which the coal price and energy-efficiency A_R are constant, and coal use R tracks Y. What is the growth rate?

ii. Discuss briefly whether or not the model provides a good heuristic explanation for why aggregate energy efficiency has risen so little in the real global economy.

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3. "To the extent that it is impossible to design around or find substitutes for expensive natural resources, the prices of commodities that contain a lot of them will rise relative to the prices of other goods and services that don't use up a lot of resources. Consumers will be driven to buy fewer resource-intensive goods and more of other things."[3]

(a) What has in fact happened to the consumption rate of energy-intensive goods (compared to GDP growth) over the last 150 years?

What has caused this change, changes in relative prices or something else? Discuss theory and, if you can, evidence.

(b) Discuss the extent to which consumers' changing consumption patterns in the future may make the transition to a climate-friendly global economy harder to achieve, rather than easier. Does it matter if these changes are driven by income effects or substitution effects?

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[3] Robert Solow, Challenge, 1973, p.47.

4. Assume an economy controlled by a social planner with a single final good produced in quantity Y using inputs of labour L and electricity E. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the resource has a small factor share). A_L and L grow exogenously at constant rates. Electricity E is produced using coal X_1, and we choose units such that

$$E = X_1,$$

i.e. the flow of energy is equal to the flow of coal. The extraction cost of coal, w_1, is constant. Furthermore, burning a unit of coal leads to ϕ units of polluting emissions,

$$D = \phi X_1.$$

Utility U is production Y minus total extraction costs, $w_1 X_1$, so

$$U = (A_L L)^{1-\alpha} E^{\alpha} (1 - \psi D) - w_1 X_1.$$

(a) i. Write down an expression for utility in terms of X_1, and find an expression for $\partial U / \partial X_1$.

ii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very small. (Hint: What does this imply about pollution damages per unit of X_1, compared to extraction costs?)

iii. Find an approximate expression for the planner's optimal choice of X_1 assuming that $A_L L$ is very large.

iv. Describe the development of the economy over time assuming that at $t = 0$, $A_L L$ is very small.

(b) Explain carefully what difference it would make if there were an alternative method of producing electricity using an input X_2 that was more expensive ($w_2 > w_1$) but emissions-free.

(c) Discuss the relevance of the model to explaining and predicting real world phenomena.