Department of Economics
Rob Hart

January 2018

Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

---

1. (a) Compare the following two models in the following three respects:

    i. How Y, R, and w_R (the resource price) develop in the long run (assuming balanced growth in the second model);

    ii. How well these model results match global aggregate observations of Y, R, and w_R for resources such as metals and fossil fuels;

    iii. The models' ability to *explain* (rather than just match) historical observations.

    - Model 1 (a variation on the 'limits to growth' model).

    $$\begin{aligned} Y_t &= \min\{A_{Lt}L_t, A_{Rt}R_t\}; \\ \dot{A}_L/A_L &= g; \\ \int_{t=0}^{\infty} R_t \mathrm{d}t &\leq S_0. \end{aligned}$$

    Labour L is fixed, and hired on perfect markets, whereas the resource R is free to extract and of 'open access' character, i.e. no individual or group has property rights over the resource (and it is not storable after extraction). A_R is constant.

    - Model 2 (a variation of the DHSS model with a resource in infinite supply but costly to extract, and competitive markets).

    $$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^{\alpha} (A_R R)^{\beta}; \\ \dot{A}_L/A_L &= g; \\ \dot{A}_R/A_R &= g_R; \\ \dot{K} &= s(Y - X) - \delta K; \\ C &= (1-s)(Y - X) \\ R &= \phi X. \end{aligned}$$

   (b) Neither of the models is much use for predicting the future development of the global economy, partly because they do not include any of "Solow's three mechanisms". Discuss how model 2 might be extended to include these mechanisms, while also assuming a more realistic (but still very simple) model of resource stocks. What difference would this make?

---

2. Assume an economy on an island with a single product, hammers. The production function is CES, with inputs of labour L_Y and iron R, with factor-augmenting technology levels A_L and A_R.[1] It can be written

$$Y = [(A_L L_Y)^{\epsilon} + (A_R R)^{\epsilon}]^{1/\epsilon}.$$

The price of hiring labour is normalized to A_L (so $w_L = A_L$). Meanwhile, iron is extracted from infinite homogeneous stocks by firms with the extraction function[2]

$$R = \phi A_L L_R.$$

[1] Units: hammers wkr^{-1} year^{-1} and hammers ton-of-iron^{-1}.

[2] So total labour L is divided between production and extraction. Later on we add research labour too.

(a)
   i. Set up the representative hammer-producer's static profit-maximization problem, and use it to derive an expression for the relative factor shares of labour and iron in terms of A_L, A_R, L_Y, and R.
   ii. Multiply each side of this expression by $(w_L L/(w_R R))^{-\epsilon}$, and rearrange, in order to obtain an expression for the relative factor shares of labour and iron in terms of A_L, A_R, w_L, and w_R.
   iii. Find the price of iron w_R, which is equal to the unit cost of extraction of iron, and recall that $w_L = A_L$. Substitute in to your answer to part (ii) above, to obtain an expression for the relative factor shares in terms of A_R and parameters ϕ and ϵ.
   iv. Assume that $\phi = 81$ and $\epsilon = -1$, while $A_L = 1$, $A_R = 1$, and $L_Y = 100$. You should find that factor share of labour is 90 percent. Find R, iron extraction. And find Y, hammer production.

(b) Assume that 10 islanders work in the research sector, and that the islanders' knowledge production functions are as follows, where z_l and z_r are measures of research effort (in researcher–years) on labour-augmenting and iron-augmenting knowledge:

$$A_{Lt+1} = A_{Lt}(0.998 + 0.002 z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(0.998 + 0.002 z_{rt+1}).$$

Furthermore, assume that relative investments z_l/z_r in period $t+1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t. (So you do not need to set up and solve the full dynamic problem.)

   i. What are relative factor shares in year $t+1$? And by how much (in percent) does R, iron extraction, grow, assuming L_Y is unchanged (i.e. $L_Y = 100$)?
   ii. Characterize the development of the economy in the long run.
   iii. Explain what happens if, instead, iron starts to run out and its price starts to increase at a constant rate. Comment briefly.

---

3. "To the extent that it is impossible to design around or find substitutes for expensive natural resources, the prices of commodities that contain a lot of them will rise relative to the prices of other goods and services that don't use up a lot of resources. Consumers will be driven to buy fewer resource-intensive goods and more of other things."[3]

   (a) What has in fact happened to the consumption rate of energy-intensive goods (compared to GDP growth) over the last 150 years?

   What has caused this change, changes in relative prices or something else? Discuss theory and, if you can, evidence.

   (b) Discuss the extent to which consumers' changing consumption patterns in the future may make the transition to a climate-friendly global economy harder to achieve, rather than easier. Does it matter if these changes are driven by income effects or substitution effects?

---

[3] Robert Solow, Challenge, 1973, p.47.

4. Assume an economy with competitive markets with a single final good produced in quantity Y using inputs of labour L and resources R. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha}(1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the pollutant has a small factor share). A_L and L grow exogenously at constant rates. Resources R can be produced a combination of two inputs X_i where $i = 1, 2$. The inputs are perfect substitutes, and

$$R = \sum_i X_i.$$

The inputs differ in two respects. Firstly, the costs of extraction w_i differ. Costs are constant for each input, hence they have constant prices, but $w_2 = (1+\gamma)w_1$, where $\gamma > 0$. Secondly, input 1 leads to polluting emissions D, according to the following equation:

$$D = X_1,$$

whereas the more expensive input does not cause any emissions. Utility U is production Y minus total extraction costs, $\sum_i w_i X_i$.

(a) i. Find an expression for MC_1, the marginal social cost of using input X_1, in terms of exogenous factors and R.
 ii. Find a corresponding expression for MC_2.
 iii. Find expressions for MB_1 and MB_2, the marginal social benefits of using the respective inputs. Comment briefly.

(b) i. Find a condition for $MC_1 = MC_2$ in terms of R and $A_L L$, and explain what it implies about the switch from input X_1 to X_2.
 ii. Describe the path of economic development in this economy as A_L and L grow (starting from a low level), assuming that the economy is optimally regulated.

(c) Discuss the relevance of the model to understanding patterns of polluting emissions over time in real economies.