Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. (a) Compare the following two models in (i) their ability to explain historical data about resource extraction rates and resource prices, and (ii) their ability to help us predict the future effects of resource scarcity.

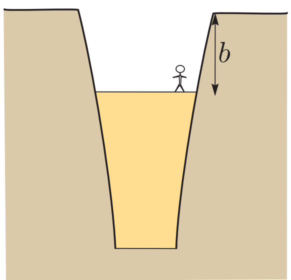
   - Model 1 (the standard DHSS model with competitive markets):

$$
\begin{aligned}
Y &= (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta}, \\
\dot{A}/A &= g, \\
\dot{K} &= sY - \delta K, \\
C &= (1-s)Y, \\
S &\geq \int_0^{\infty} R_t \mathrm{d}t.
\end{aligned}
$$

   - Model 2 (the DHSS model with a resource in infinite supply but costly to extract, and competitive markets):

$$
\begin{aligned}
Y &= (A_L L)^{1-\alpha-\beta} K^{\alpha} R^{\beta}; \\
\dot{A}_L/A_L &= g; \\
\dot{K} &= s(Y - X) - \delta K; \\
C &= (1-s)(Y - X) \\
R &= \phi X.
\end{aligned}
$$

   (b) Discuss an extension to the latter model in which we instead assume that the resource stock is finite and inhomogeneous. More specifically, assume many (competitive) resource owners who each own a stock which looks something like that in the picture below, where b indicates the depth of the stock, and unit extraction costs increase in b. Can this model help us to both explain and predict resource trends?



2. [A]s the earth's supply of particular natural resources nears exhaustion, and as natural resources become more and more valuable, the motive to economize those natural resources should become as strong as the motive to economize labor. The productivity of resources should rise faster than now—it is hard to imagine otherwise.

[Solow, *Is the end of the world at hand?*, Challenge, 1973, p47.]

(a) Between 1800 and 1973 the price of primary energy fell compared to the price of labour.

  i. Explain why, in theory, this might lead labour-augmenting knowledge to grow faster than energy-augmenting knowledge.

  ii. Discuss evidence.

(b) Over the next 50 years there is likely to be a global switch from fossil to renewable sources of energy. Will this switch lead to rapid increases in the efficiency of renewable energy technologies, and hence declines in the price of (for instance) electricity generated from renewable sources? Will electricity from renewable sources become cheaper than electricity from fossil, without the need for carbon taxes? Discuss theory and evidence.

3. Assume an economy in which there are three final goods, heating, transport, and other goods. There is no capital and no investment: people already have houses (which need heating) and cars (which need driving). Houses are heated with coal, which is cheap. Cars are driven with gasoline, which is expensive. Other goods are produced using pure labour, and are either consumed or used to extract gasoline and coal. All markets are perfect. The question is about what happens when car efficiency increases.

   The production function for other goods Y is:

$$Y = A_L L,$$

   where A_L is labour productivity and L is labour. Consumption C is

$$C = Y - (X_c + X_g), \tag{1}$$

   where X_c and X_g are inputs of final goods into the extraction of coal and gasoline. Coal c and gasoline g are extracted as follows:

$$Q_c = X_c/\phi_c \qquad \text{and} \qquad Q_g = X_g/\phi_g,$$

   where the units of both Q_c and Q_g are megajoules (i.e. energy). Heat H and transport T are produced as follows:

$$Q_H = A_c Q_c \qquad \text{and} \qquad Q_T = A_g Q_g.$$

   The utility function is

$$U = \min(Q_H/\gamma_H, Q_T/\gamma_T, C).$$

   Labour L and all parameters and productivities are exogenously given.

   (a) i. Find an expression for X_c in terms of U, parameters, and exogenous variables.[1]

   ii. Find a corresponding expression for X_g.

   iii. Find expressions for energy use Q_c and Q_g.

   (b) Now use equation 1 and your expressions for X_c and X_g to find an expression for U in terms of exogenous quantities.

   (c) Assume the following values for the parameters and exogenous variables in the model.

ϕ_c	ϕ_g	γ_H	γ_T	A_L	A_c	A_g	L
0.0125	0.05	4	1	1	1	1	1

   i. What are the factor shares of coal and gasoline?[2]

   ii. How much energy is consumed in the respective sectors?

[1]Hint: Use the Leontief utility function to write down an equation linking Q_H to U. Then substitute for Q_H and rearrange.

[2]Hint: Total factor expenditures are equal to the final goods devoted to their extraction, whereas GDP$= C = U$.

(d) Assume that the efficiency of car engines doubles exogenously, so $A_g = 2$.

  i. What would a naive ‘engineering’ analysis predict about energy savings in the economy?

  ii. What are actual energy savings in the economy?

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4. Assume an economy with competitive markets with a single final good produced in quantity Y using inputs of labour L and resources R. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the pollutant has a small factor share). A_L and L grow exogenously at constant rates. Resources R can be produced a combination of two inputs X_i where $i = 1, 2$. The inputs are perfect substitutes, and

$$R = \sum_i X_i.$$

The inputs differ in two respects. Firstly, the costs of extraction w_i differ. Costs are constant for each input, hence they have constant prices, but $w_2 = (1 + \gamma) w_1$, where $\gamma > 0$. Secondly, input 1 leads to polluting emissions D, according to the following equation:

$$D = X_1,$$

whereas the more expensive input does not cause any emissions. Utility U is production Y minus total extraction costs, $\sum_i w_i X_i$.

(a) i. Find an expression for MC_1, the marginal social cost of using input X_1, in terms of exogenous factors and R.

  ii. Find a corresponding expression for MC_2.

  iii. Find expressions for MB_1 and MB_2, the marginal social benefits of using the respective inputs. Comment briefly.

(b) i. Find a condition for $MC_1 = MC_2$ in terms of R and $A_L L$, and explain what it implies about the switch from input X_1 to X_2.

  ii. Describe the path of economic development in this economy as A_L and L grow (starting from a low level), assuming that the economy is optimally regulated.

(c) Discuss the relevance of the model to understanding patterns of polluting emissions over time in real economies.
