Brief suggested answers

EGSD Examination, January 2016.

Note that longer answers may be required for full marks. For instance, it is important to show your working in calculation questions. And for discussion or essay questions my answers are intended as an outline.

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1. (a) Equation 1 is a Leontief production function with inputs of labour and resources, augmented by productivity factors. So there is no substitutability between inputs in the short run.

   Equation 2 shows that labour productivity grows exogenously at a constant rate. Since the labour force L is constant, this implies that $A_L L$ rises, hence driving up firms' demand for the resource R (since A_R is constant).

   Equation 3 shows that we have a non-renewable resource. Since it is open access and free to extract, the resource price will be zero, and firms will extract whatever resources they need to maintain production.

   (b) The two cases are shown in the tables below. Resources are 'sucked in' to the production process up to the time at which they run out, when nothing more can be produced. Growth in A_R delays this time. (Note: You don't need to show all these figures to get full marks, but some calculations are necessary to show that you understand fully how the economy works.)

t	0	1	2	3	4	5	6
$A_{Lt}L$	1	2	4	8	16	32	64
Case (i)							
S_t	10	9	7	3	0	0	0
A_{Rt}	1	1	1	1	1	1	1
R_t	1	2	4	3	0	0	0
Y_t	1	2	4	3	0	0	0
Case (ii)							
S_t	10	9	8	6·67	4·67	1·33	0
A_{Rt}	1	2	3	4	5	6	7
R_t	1	1	1·33	2	3·33	1·33	0
Y_t	1	2	4	8	16	8	0

   (c) The model as it stands may be of some help with regard to historical observations, but of little relevance for predicting the future.

   Regarding historical observations the model with constant A_R predicts a zero resource price and exponentially increasing extraction, tracking growth in global product. In fact we observe resource prices constant but positive (caused by a combination of extraction costs and in some cases market power), with extraction tracking growth. However, the rise in R does not seem to be driven by a failure of A_R to rise, rather it is driven by a shift in consumption patterns towards resource-intensive products.

Regarding the future, even if non-renewable resources were open access in the distant past, we know that as they become valuable access to them is restricted and the price mechanism kicks in. So if resources really are running out then they will become very expensive.

Given an increase in the resource price, we expect to see adaptations of the type described by Solow (1973): shifts in consumption patterns, endogenous increases in A_R, and the endogenous development of alternative resource inputs. The model therefore needs to be extended in several directions if it is to help us understand the possibilities and limitations of the adaptation process. Furthermore, we need an extraction model to predict future resource prices in the first place, we should not simply assume that they will rise.

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2. (a) Total GDP is 9 houses per year, so per capita we have 9/11 houses per person per year. Relative prices and relative shares are equal: $w_l/w_r = 9 = w_lL/(w_rR)$.

   (b) i. The key here is to understand that total labour in research is 1, so we have $z_l = 0.9$ and $z_r = 0.1$. If follows that both A_L and A_R rise by a factor of 1.09, and GDP and GDP per capita rise by the same factor, while relative prices and factor shares are unchanged.

   ii. If the flow of trees diminishes towards zero over time, the factor share of trees tends to rise, driving investment in tree-augmenting knowledge A_R, and compensating for the decline in the physical flow. (But if the flow drops to zero then there can be no production.)

   iii. A_L is labour productivity, so it is the rate at which workers can cut up trees if trees are abundant. It might increase if the penknives are replaced by saws, or even a sawmill. A_R is the productivity of trees, so it is the number of houses that can be made out of a tree if labour is abundant. It might rise if the islanders work out a new way of cutting up trees to get out more planks.

   (c) In the model it is very easy to raise the productivity of trees, so if the flow of trees declines we simply make our houses using fewer trees (after investing in the technology which allows us to do so). In the real economy it may also be possible to economize on the resource and energy flows needed to make given products —consider the use of energy to make artificial light or motive power—but there are typically limits to this process. On the other hand, there are other ways to adapt in the real economy, such as using other inputs to make houses (or whatever the final product might be), or shifting consumption patterns away from final products which are resource- and energy-intensive. These mechanisms are in the long run likely to be at least as important as resource efficiency.

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3. (a) i. $\alpha/(1-\alpha)$.

   ii. We know from (i) that $p_1 Y_1 = \alpha Y$, and $p_2 Y_2 = (1-\alpha)Y$. Use the second of these to show that

   $$p_2 = (1-\alpha)\left(\frac{a_l L}{a_r R}\right)^{\alpha}.$$

   But $p_2 = w_r/a_r$, so we can rearrange to find

   $$R = a_l L(1-\alpha)^{1/\alpha} a_r^{(1-\alpha)/\alpha} w_r^{-1/\alpha}.$$

   Finally note that we know that $w_r = 1$.

   iii. Raising a_r raises total factor productivity in the economy and causes an increase in energy consumption—backfire—whereas raising w_r has a strong negative effect on energy consumption, since it causes consumers to reduce their consumption of the energy-intensive good.

   (b) The model is not very relevant when energy-intensive products have a much lower energy share than 100 percent, since substitution towards such products will not cause nearly such a large rebound effect, while rises in the price of energy will not have such a large negative effect on their consumption either. To explain the rise in global energy use despite increases in a_r we need not only substitution effects of the type which are in the model above, but also income effects: as incomes rises, consumers choose more energy-intensive consumption types.

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4. See notes and the book for this question. Very briefly ...

   (a) The key to understanding resource prices is to see that stocks are typically very large so scarcity rents are not significant, and technological progress and rising input prices tend to cancel each other out (why?), while increasing 'depth' only has a rather small effect. These trends are likely to continue for some time into the future. To predict trends for individual resources we need to know about the nature of stocks (are they about to run out, so scarcity will become an issue, or is depth about to start increasing rapidly), and what substitutes are available (if there are close, abundant substitutes then the price will not rise above the price of the substitutes even if the resource is running out).

(b) In the very long run the rate of use of energy, land, and resources must level out. Given a constant rate of use, we expect their prices to rise at the overall growth rate.

Consider the Cobb–Douglas aggregate function (without capital, for simplicity), which fits the aggregate data remarkably well:

$$Y = A_L^{1-\alpha} R^{\alpha}.$$

Then given perfect markets $w_r = \alpha Y/R$, and if Y keeps growing (because A_L grows) while R is constant, then w_r must grow at the same rate as Y.