Department of Economics
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Examination

Economic Growth and Sustainable Development, NA0167.

Rules

Permitted aids: Pen, paper, and pocket calculator (provided).

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Directed technological change and sustainability.
3. Consumption, rebound, and sustainability.
4. Any or all of the above.

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1. (a) Consider the DHSS model with competitive markets, zero extraction costs, a fixed saving rate, and constant population:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta, \\ \dot{A}_L/A_L &= g, \\ \dot{K} &= sY - \delta K, \\ C &= (1-s)Y, \\ S &\geq \int_0^\infty R_t \mathrm{d}t. \end{aligned}$$

The market interest rate is constant, denoted ρ.

i. Explain intuitively how the resource price grows in this model economy.

ii. Given the Cobb–Douglas final-good production, find an equation linking $\dot{R}/R$ to $\dot{Y}/Y$ and ρ.

(b) Assume instead a resource in infinite supply but costly to extract:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\alpha R^\beta; \\ \dot{A}_L/A_L &= g; \\ \dot{K} &= s(Y - X) - \delta K; \\ C &= (1-s)(Y - X) \\ R &= \phi X. \end{aligned}$$

i. Explain intuitively how the resource price grows in this model economy.

ii. Given the Cobb–Douglas final-good production, find an equation linking $\dot{R}/R$ to $\dot{Y}/Y$.

(c) Compare the models in (i) their ability to explain historical data about resource extraction rates and resource prices, and (ii) their ability to help us predict the future effects of resource scarcity.

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2. Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour L and trees R, with factor-augmenting technology levels A_L and A_R.[1] It can be written

$$Y = [(A_L L)^\epsilon + (A_R R)^\epsilon]^{1/\epsilon}.$$

The parameter $\epsilon = -1$. There are 22 people on the island, of whom 20 work in production and 2 in research, and 10 trees/year wash up on the shore. All markets are perfect.

(a) Assume that—in year t—the islanders have a technology called 'saws' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $A_L = 0.5$, $A_R = 1$. Calculate total GDP, and the relative factor shares of labour and trees.

[1] Units: houses wkr^{-1} year^{-1} and houses tree^{-1}.

(b) Assume that the islanders' knowledge production functions are as follows, where z_l and z_r are measures of research effort (in researcher–years) on labour-augmenting and tree-augmenting knowledge:

$$A_{Lt+1} = A_{Lt}(1 + 0.02z_{lt+1});$$
$$A_{Rt+1} = A_{Rt}(1 + 0.02z_{rt+1}).$$

Furthermore, assume that relative investments z_l/z_r in period $t + 1$ are equal to relative factor shares $w_l L/(w_r R)$ in period t.

i. What are GDP and relative factor shares in year $t+1$, if both the flow of trees and the labour force remain constant?

ii. What are GDP and relative factor shares in year $t + 2$ if the flow of trees drops to 1?

iii. Describe how the economy develops after period $t + 2$ if $L = 20$ and $R = 1$.

(c) Assume a real, modern economy in which an important natural resource starts to run out (or can no longer be used because of pollution problems). Describe mechanisms through which the economy is likely to adapt, giving concrete examples to back up theoretical arguments. How important is the mechanism of the model above?

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3. Discuss the following statement.

Changing consumption patterns are the cause of the rapid growth of global primary energy use illustrated in Figure 1. This implies that rebound effects are very powerful, hence increases in energy efficiency will not on their own reduce energy consumption.

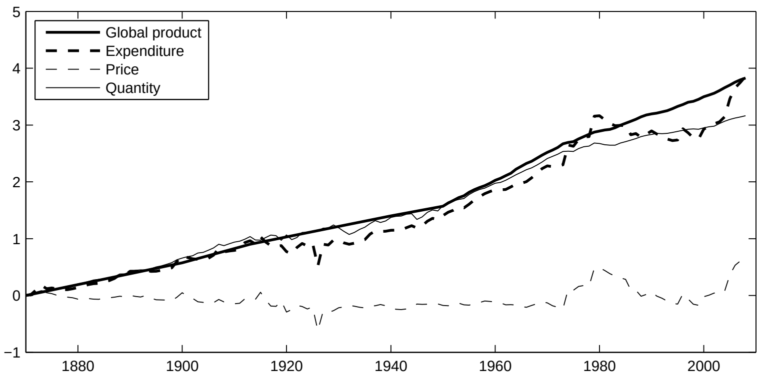

Figure 1: Long-run growth in global production and primary energy use. Natural log scale.[2]

[2]Energy: Coal, oil, natural gas, and biofuel. For data sources see Economic Growth and Sustainable Development, Hart (2016).

4. Assume an economy with competitive markets with a single final good produced in quantity Y using inputs of labour L and resources R. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the pollutant has a small factor share). A_L and L grow exogenously at constant rates. Resources R can be produced a combination of two inputs X_i where $i = 1, 2$. The inputs are perfect substitutes, and

$$R = \sum_i X_i.$$

The inputs differ in two respects. Firstly, the costs of extraction w_i differ. Costs are constant for each input, hence they have constant prices, but $w_2 = (1 + \gamma)w_1$, where $\gamma > 0$. Secondly, input 1 leads to polluting emissions D, according to the following equation:

$$D = X_1,$$

whereas the more expensive input does not cause any emissions. Utility U is production Y minus total extraction costs, $\sum_i w_i X_i$.

(a) i. Find an expression for MC_1, the marginal social cost of using input X_1, in terms of exogenous factors and R.

ii. Find a corresponding expression for MC_2.

iii. Find expressions for MB_1 and MB_2, the marginal social benefits of using the respective inputs. Comment briefly.

(b) i. Find a condition for $MC_1 = MC_2$ in terms of R and $A_L L$, and explain what it implies about the switch from input X_1 to X_2.

ii. Describe the path of economic development in this economy as A_L and L grow (starting from a low level), assuming that the economy is optimally regulated.

(c) Discuss the relevance of the model to understanding patterns of polluting emissions over time in real economies.