

An illustration of a landscape with a brown ground, a green tree on the right, and a small orange figure in the center. A large, light-colored, teardrop-shaped cloud or smoke plume is positioned above the ground. The text "Sustainable Development" is written in white on a brown background at the bottom.

Sustainable Development

An identical illustration to the one on the left, featuring a landscape with a brown ground, a green tree, a small orange figure, and a large light-colored cloud. The text "Sustainable Development" is written in white on a brown background at the bottom.

Sustainable Development

## Chapter 3

---

# Post-Malthusian—industrial—growth

---

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

# The Solow and Ramsey models

# What is a production function?

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

# What is a production function?

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

Compare the following equations. Discuss similarities and differences.



$$\Delta G = \Delta H - T \Delta S_{int};$$

$$Y_i = \min(A_L l_i, A_K k_i, A_R r_i).$$

# What is a production function?

The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

Vintage growth models

Endogenous growth

Consider the following equations.

$$Y = A_L L,$$

$$Y = F(A_L L, A_K K),$$

$$Y = F(A_L L, A_K K, A_R R).$$

Suggest units for each of the variables.

What drives growth in  $Y$ ? And  $Y/L$ ?

# Neoclassical growth models

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

# Neoclassical growth models

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

The production function:

$$Y = F(A_K K, A_L L).$$

$$F'_K, F'_L \geq 0.$$

$$F''_K, F''_L \leq 0.$$

# Neoclassical growth models

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

Technology:

Nonrival. Nonexcludable. Exogenous.



# Neoclassical growth models

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

$$K_{t+1} = (1 - \delta)K_t + sY_t,$$

$$K_{t+1} = (1 - \delta)K_t + sF(K_t, L_t).$$

# Neoclassical growth models

The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

Vintage growth models

Endogenous growth

Consider Figure 1. What are the values of  $K$ ,  $L$ ,  $Y$ , and  $s$ ? And  $\delta$ ?

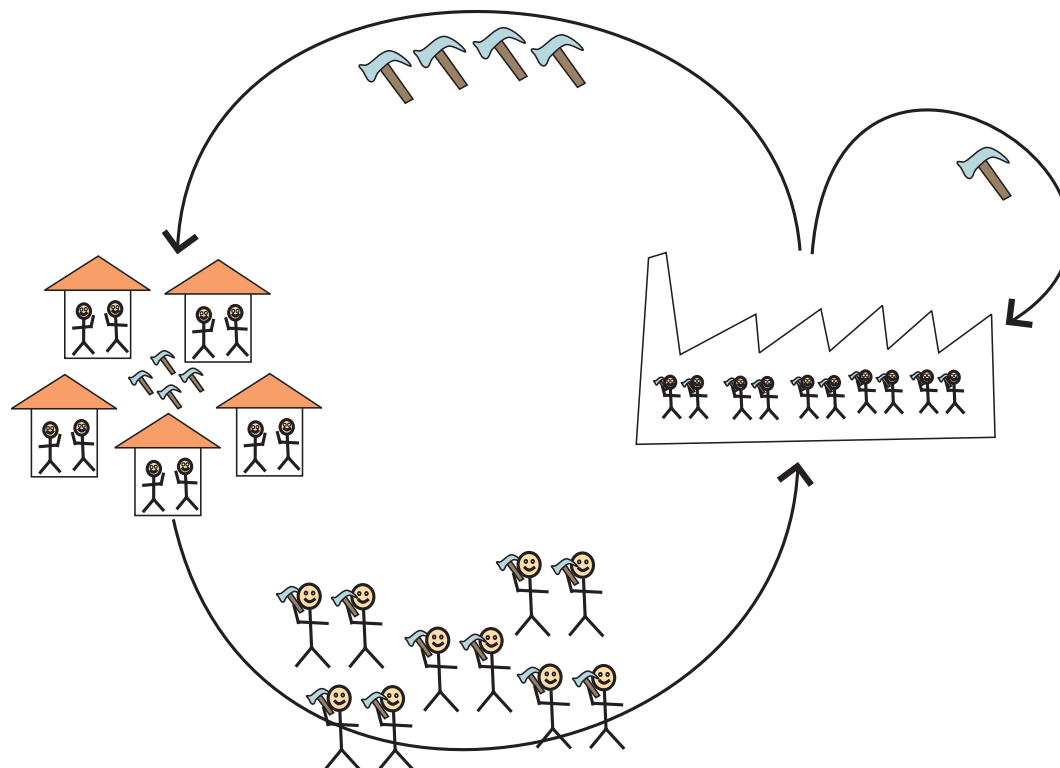


Figure 1: A Solovian economy in which hammers are the final good.

# Solow (with constant technology)

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
  - Solow (with constant technology)
  - Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

# Solow (with constant technology)

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
  - Solow (with constant technology)
  - Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

$$K_{t+1} = (1 - \delta)K_t + sY_t.$$

$$Y = (A_L L)^{1-\alpha} K^\alpha.$$

Cobb–Douglas properties? Unit elasticity of substitution!

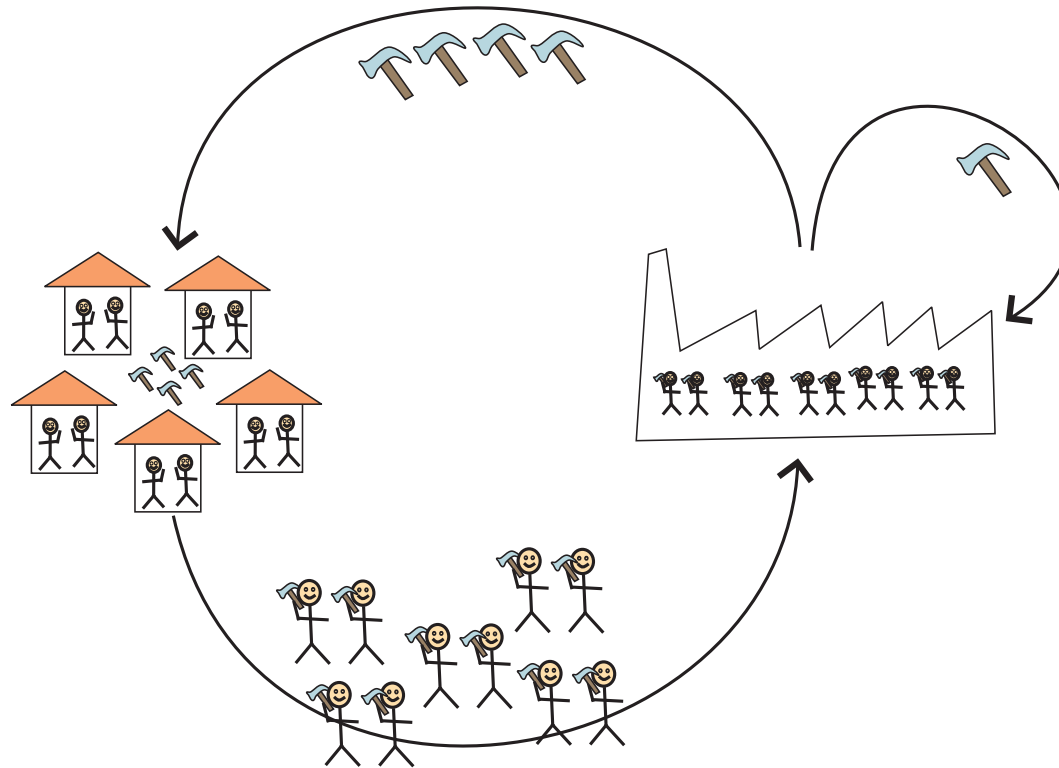
# Solow (with constant technology)

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth



Solve the model when  $\alpha = 1/3$  and  $\delta = 0.1$ .

Characterize the long-run steady state.

## Solow (with constant technology)

The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
  - Solow (with constant technology)
  - Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

Vintage growth models

Endogenous growth

Discuss effects of different saving rates across countries.

And if there is a global capital market?

# Solow with exogenous growth in $A_L$

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

## Solow with exogenous growth in $A_L$

The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

Vintage growth models

Endogenous growth

$$Y = (A_L L)^{1-\alpha} K^\alpha$$

$$\dot{A}_L/A_L = g_{A_L}$$

$$\dot{L}/L = n$$

$$\dot{K} = sY - \delta K.$$

On b.g.p.,  $\dot{K}/K$  constant, hence  $sY/K - \delta$  constant, hence  $Y/K$  constant, hence  $(A_L L/K)^{1-\alpha}$  constant, and

$$\dot{K}/K = \dot{A}_L/A_L + \dot{L}/L.$$

$y = Y/L$ . What is  $\dot{y}/y$ ?



## Solow with exogenous growth in $A_L$

### The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

### Vintage growth models

### Endogenous growth

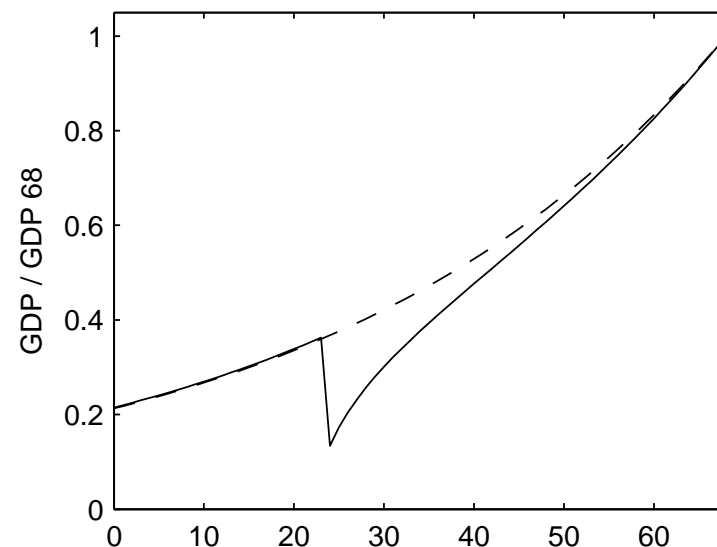
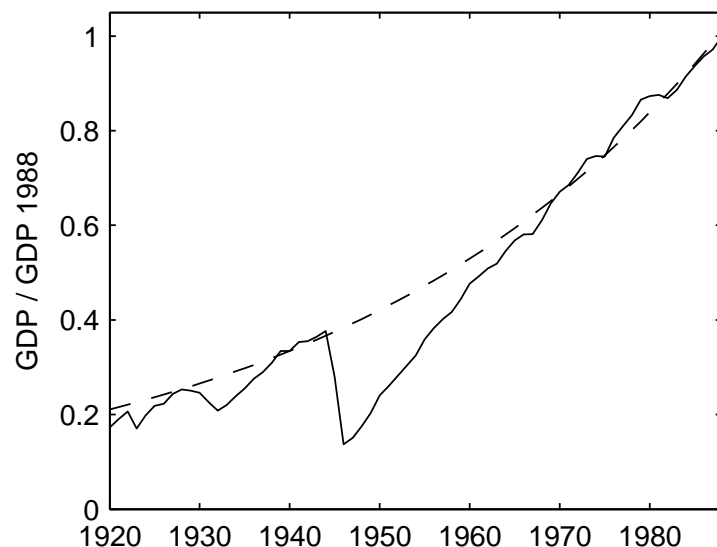


Figure 2: Testing the Solow model. Upper panel: German real GDP, compared to an estimated constant-growth trend (growth rate 2.3 percent per year). Lower panel: Model simulation assuming the same growth trend in labour productivity and parameters  $\alpha = 0.3$ ,  $\delta = 0.1$ ,  $s = 0.2$ , with a massive shock to capital (the stock is divided by a factor of 30 at the end of year 24).

# Solow with exogenous growth in $A_L$

The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

Vintage growth models

Endogenous growth

What is the Solow model good for?

What drives growth in the Solow model?

What drives growth in real economies?

# Ramsey

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

# Ramsey

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

where

$$\max \sum_h U^h,$$

$$U^h = \sum_t u^h(c_t) \beta^t.$$

and

$$u = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

What is the Ramsey model good for?

# No capital

## The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

## Vintage growth models

## Endogenous growth

## No capital

### The Solow and Ramsey models

- What is a production function?
- Neoclassical growth models
- Solow (with constant technology)
- Solow with exogenous growth in  $A_L$
- Ramsey
- No capital

### Vintage growth models

### Endogenous growth

$$Y = A_L L [K / (A_L L)]^\alpha,$$

and note that in balanced growth  $K$  grows at the same rate as  $A_L L$ , so  $K / (A_L L)$  is constant, then it should be clear that in long-run analysis we lose little by simplifying the production function to

$$Y = A_L L.$$

The same conclusion applies when we include resources in the production function. Then we have (for instance)

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^\beta R^\alpha \\ &= (A_L L)^{1-\alpha} [K / (A_L L)]^\beta R^\alpha. \end{aligned}$$

What do we lose by dropping capital? Gain?

The Solow and Ramsey models

Vintage growth models

- A simple model

Endogenous growth

# Vintage growth models

# A simple model

The Solow and Ramsey models

Vintage growth models

- A simple model

Endogenous growth



The Solow and Ramsey models

Vintage growth models

• A simple model

Endogenous growth

## A simple model

One person per machine. Machines last for ten years, i.e. 1 period.

Year 2000, 100 people and 100 (new) machines. Of these, 80 people—each with a machine—work on the production of consumer goods, while 20 people—each with a machine—work on the manufacture of the next-generation machines.

100 new machines are ready in 2010, and they are 20 percent more productive than the old.

Period-1 wage: 100 crowns per period.

Interest rate per period of 10 years is 100 percent.

What is GDP in period 1? What is the labour share, and what is the capital share?

## A simple model

The Solow and Ramsey models

Vintage growth models

• A simple model

Endogenous growth

As above, but open economy far behind the technology frontier.

Produces and exports consumption goods, imports machines. Higher quality machines yield higher GDP per capita, but cost more.

Each quality step gives 20 percent higher GDP, and the machines to achieve it cost 20 percent more.

GDP per capita 10 000 USD per period, and for 2000 USD per capita the country can upgrade—in the next 10-year period—to machines that yield 12 000 USD per capita per period.

The country invests in new machines that are five rungs up the ladder compared to its current machines. What is investment as a proportion of GDP? Consumption? Growth rate?

The Solow and Ramsey models

Vintage growth models

Endogenous growth

- Endogenous growth

# Endogenous growth

# Endogenous growth

The Solow and Ramsey models

Vintage growth models

Endogenous growth

- Endogenous growth

$$A_{Lt+1} = A_{Lt}[1 - \delta + \zeta(\Omega L_t)^\phi],$$

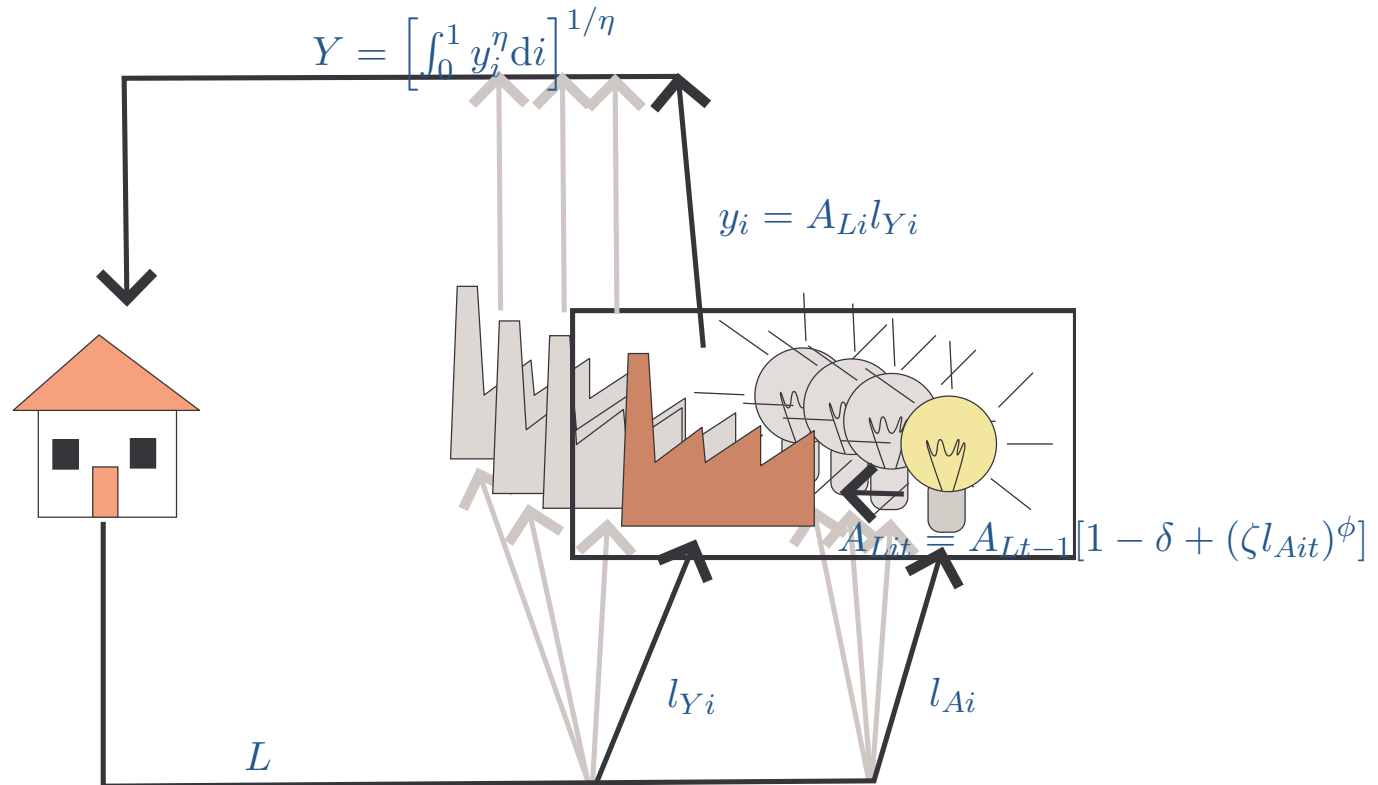
# Endogenous growth

The Solow and Ramsey models

Vintage growth models

Endogenous growth

- Endogenous growth



Single firm's Lagrangian?  $MR = \eta p_i$ . FOCs in  $L_Y$ ,  $L_A$ , and  $A_{Lit}$ .

Investment in knowledge?

# Endogenous growth

The Solow and Ramsey models

Vintage growth models

Endogenous growth

- Endogenous growth

We can take an 'exogenous growth model' in which workers devote a fixed proportion of their time to research for no obvious reason, and turn it into an endogenous growth model in which time spent on research is the result of competing firms solving optimization problems. For our purposes, the key use of the endogenous model will be in later chapters when we wish to analyse how firms choose between investment in alternative types of technology, such as 'clean' and 'dirty'.