Sustainable Development

Part 7

Substitution between alternative resource inputs

A simple model with alternative resource inputs

A simple model with alternative resource inputs

Recall: $$Y = (A_L L)^{1-\alpha}(A_R R)^{\alpha}.$$

$$\max \pi = p_y (A_L L)^{1-\alpha}(A_R R)^{\alpha} - w_l L - w_r R;$$

$$w_r R = \alpha Y.$$

Now $$R_t = \left[(\gamma_c A_{ct} X_{ct})^{\epsilon} + (\gamma_d A_{dt} X_{dt})^{\epsilon}\right]^{1/\epsilon}.$$

Assume $A_c = A_d = A$, and fix $A_R = 1$.

$$Y_t = (A_t L_t)^{1-\alpha} R_t^{\alpha},$$

$$R_t = A_t \left[(\gamma_c X_{ct})^{\epsilon} + (\gamma_d X_{dt})^{\epsilon}\right]^{1/\epsilon},$$

and $$C_t = Y_t - (w_{ct} X_{ct} + w_{dt} X_{dt}),$$

A simple model with alternative resource inputs

$$\pi = w_{rt} A_t \left[(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon\right]^{1/\epsilon} - w_{ct} X_{ct} - w_{dt} X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $$w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c / w_c)^{\epsilon/(1-\epsilon)}$$

and $$w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d / w_d)^{\epsilon/(1-\epsilon)}.$$

A simple model with alternative resource inputs

- A simple model with alternative resource inputs
- Technological change

$$\pi = w_{rt} A_t \left[(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon \right]^{1/\epsilon} - w_{ct} X_{ct} - w_{dt} X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $$w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c / w_c)^{\epsilon/(1-\epsilon)}$$

and $$w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d / w_d)^{\epsilon/(1-\epsilon)}.$$

So the relative factor shares of the resources are

$$\frac{w_c X_c}{w_d X_d} = \left(\frac{\gamma_c / w_c}{\gamma_d / w_d} \right)^{\epsilon/(1-\epsilon)}.$$

This implies that the resource that is cheaper per efficiency unit takes the larger factor share, and the advantage is bigger the higher is the substitutability between the resources (i.e. when $\epsilon \to 1$).

A simple model with alternative resource inputs

$$\pi = w_{rt} A_t \left[(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon \right]^{1/\epsilon} - w_{ct} X_{ct} - w_{dt} X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $$w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c / w_c)^{\epsilon/(1-\epsilon)}$$

and $$w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d / w_d)^{\epsilon/(1-\epsilon)}.$$

Because we have perfect markets, price equals unit cost so

$$\begin{aligned} w_r &= (w_c X_c + w_d X_d)/R \\ &= w_r^{1/(1-\epsilon)} (1/A) \left[(\gamma_c / w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d / w_d)^{\epsilon/(1-\epsilon)} \right] \\ &= \left\{ A / \left[(\gamma_c / w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d / w_d)^{\epsilon/(1-\epsilon)} \right] \right\}^{\epsilon/(1-\epsilon)}. \end{aligned}$$

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So we have

$$w_r = \left\{ A/[(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{\epsilon/(1-\epsilon)}.$$

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So we have

$$w_r = \left\{ A/[(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{\epsilon/(1-\epsilon)}.$$

And since $w_r R = \alpha Y$ we have

$$w_r = \alpha(AL/R)^{1-\alpha},$$

and we can eliminate w_r to yield

$$R = AL \left\{ \alpha[(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{1/(1-\alpha)}.$$

So if w_c and w_d are both constant then R grows at the same rate as Y, i.e. $g + n$, the sum of the growth rates of labour productivity and population.

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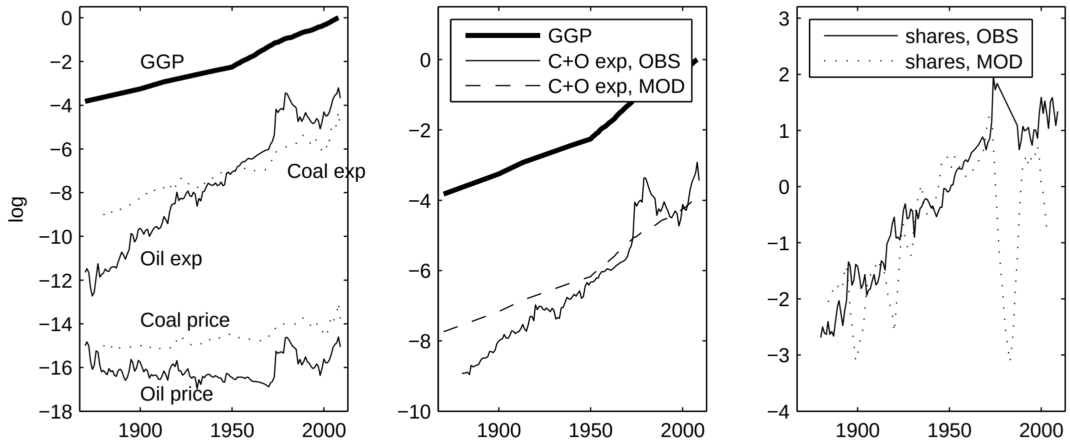

Long-run growth in prices and factor expenditure, compared to growth in global product, for crude oil and coal, and a test of the model. In the left-hand figure we see observed prices and expenditures, with expenditures compared to global product. In the middle figure we see observed total expenditure on coal and oil, compared to the model prediction (based on the prices). And in the right-hand figure we see the observed relative factor shares of coal and oil, compared to the model prediction. In the calibrated model we have $\alpha = 0.02$, $\gamma_c/\gamma_d = 0.55$, and $\epsilon = 0.76$.

A simple model with alternative resource inputs

- A simple model with alternative resource inputs
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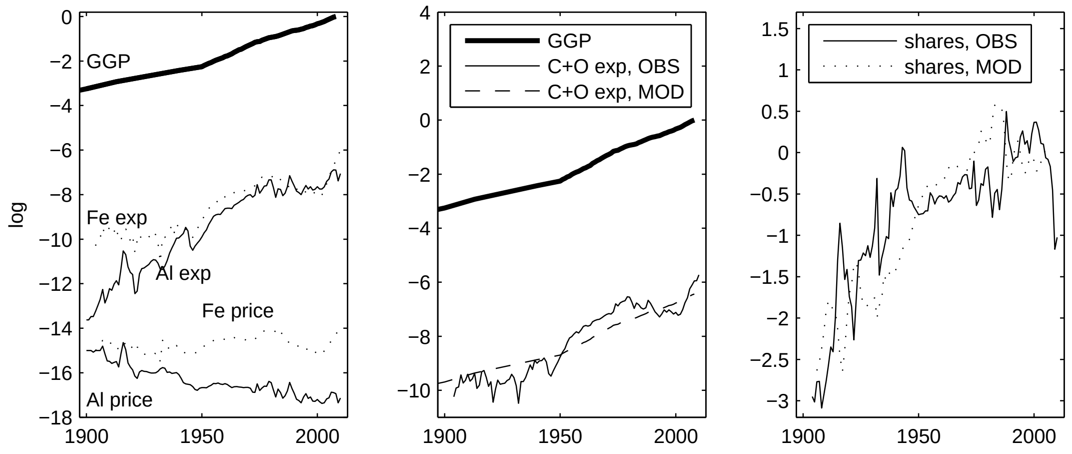

Long-run growth in prices and factor expenditure, compared to growth in global product, for iron and aluminium, and a test of the model. In the left-hand figure we see observed prices and expenditures, with expenditures compared to global product. In the middle figure we see observed total expenditure on iron and aluminium, compared to the model prediction (based on the prices). And in the right-hand figure we see the observed relative factor shares of iron and aluminium, compared to the model prediction. In the calibrated model we have $\alpha = 0.002$, $\gamma_c/\gamma_d = 50$, and $\epsilon = 0.55$.

Technological change

Technological change

Recall that relative investments are equal to relative factor shares in a model with L and R:

$$\frac{z_{lt}}{z_{rt}} = \frac{w_{lt}L_t}{w_{rt}R_t} = \left(\frac{A_{lt}L_t}{A_{rt}R_t}\right)^{\epsilon}.$$

In a model with C and D making R we have

$$\frac{z_{ct}}{z_{dt}} = \frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}C_t}{A_{dt}D_t}\right)^{\epsilon}.$$

If we add the assumption that knowledge stocks grow independently then we have

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{z_{ct}}{z_{dt}}\right)^{\phi}\left(\frac{\zeta_d}{\zeta_c}\right).$$

- A simple model with alternative resource inputs
- Technological change

Technological change

$$\frac{z_{ct}}{z_{dt}} = \frac{w_{ct} C_t}{w_{dt} D_t} = \left(\frac{A_{ct} C_t}{A_{dt} D_t} \right)^{\epsilon}.$$

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{z_{ct}}{z_{dt}} \right)^{\phi} \left(\frac{\zeta_d}{\zeta_c} \right).$$

Now assume a b.g.p. on which relative prices are exogenous and constant. Then z_c/z_d must be constant, and also A_c/A_d. So

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = 1 = \left(\frac{w_{ct} C_t}{w_{dt} D_t} \right)^{\phi} \left(\frac{\zeta_d}{\zeta_c} \right) = \left(\frac{A_{ct} C_t}{A_{dt} D_t} \right)^{\epsilon\phi} \frac{\zeta_d}{\zeta_c}.$$

So on a b.g.p. the shares of C and D are fixed. But is the b.g.p. stable?

Technological change

Focus on intuition.

Imagine the economy is on a b.g.p., and then a small shock shifts it such that the share of C increases. What happens?

- A simple model with alternative resource inputs
- Technological change

Technological change

We have
$$\frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}C_t}{A_{dt}D_t}\right)^{\epsilon}$$

and
$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{w_{ct}C_t}{w_{dt}D_t}\right)^{\phi},$$

hence
$$\frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}}{A_{dt}}\right)^{\epsilon/(1-\epsilon)} \left(\frac{w_{ct}}{w_{dt}}\right)^{-\epsilon/(1-\epsilon)}$$

and
$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct}/w_{ct}}{A_{dt}/w_{dt}}\right)^{\epsilon\phi/(1-\epsilon)}.$$

Multiply both sides by $\left(\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}}\right)^{-\epsilon\phi/(1-\epsilon)}$ to obtain

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct-1}/w_{ct}}{A_{dt-1}/w_{dt}}\right)^{\epsilon\phi/(1-\epsilon(1+\phi))}.$$

Technological change

We have

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct-1}/w_{ct}}{A_{dt-1}/w_{dt}}\right)^{\epsilon\phi/(1-\epsilon(1+\phi))}.$$

Assume we are on a b.g.p., and let A_c rise a little due to a shock. What happens?

Is the b.g.p. stable?

Technological change

- A simple model with alternative resource inputs
- Technological change

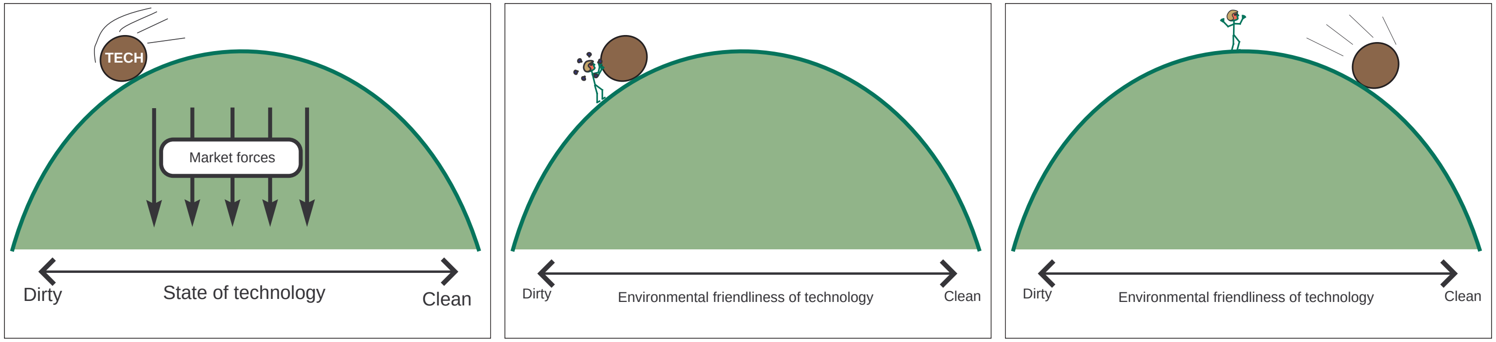

Figure 1: Illustration of how relative prices (the shape of the economic landscape) determine the relative levels of technology augmenting clean and dirty inputs in the model, and the role of a regulator.

Evidence?

What went wrong?