Sustainable Development

Exercises 2018, v.2

1. Megafaunal extinction

1.1 Assume a small group of humans settles on an island on which there is a single animal species, which the humans hunt using fixed technology. Human population evolves according to the following equation, where Y is total meat harvest, L is population, and θ_L and $\bar{y}$ are parameters:

$$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(Y_t/L_t)].$$

Meat harvest per capita is linearly related to the animal population N:

$$Y/L = \phi N.$$

Finally, the animal population evolves according to

$$N_{t+1} - N_t = \theta_N N_t [1 - N_t/N^*] - Y_t.$$

(a) Find an equation for the evolution of the human population in terms only of the animal population.

(b) Assume the existence of a steady state in which both populations are constant. Find N and L in this state, denoted N_S and L_S.

(c) Find a condition on $\overline{Y}$, ϕ, and N^* which is necessary for the existence of a steady state. Explain briefly.

(d) Outline the dynamics of the economy from the time that humans arrive.

(e) Explain why animal extinction never occurs, and what adaptations to the model might lead to animal extinction.

(f) Discuss potential lessons relevant to management of the global environment in the modern era.

1.2 Consider the economy of the previous question.

(a) Find the value of N_t which maximizes Y subject to the restriction that the animal population is constant, i.e. $N_{t+1} - N_t = 0$.

(b) Find the maximum sustainable human population, L^*, and hence also L^*/L_S (where L_S was calculated above), assuming that ϕN^* is much greater than $\bar{y}$.

(c) Assume that the economy is in the long-run laissez-faire equilibrium. Would a shift to the other equilibrium be painless?

2. Solow

2.1 (a) The general production function in the Solow model is

$$Y = F(K, L, A).$$

State and explain the key assumptions about the properties of this production function.

(b) What is capital in the model, and how does it accumulate?

(c) The representative firm's problem can be written as follows.

$$\max_{K\geq 0, L\geq 0} F(K,L,A) - RK - wL.$$

What are R and w (in words)? Find expressions for them in terms of the marginal products of the inputs, assuming that firms are price takers.

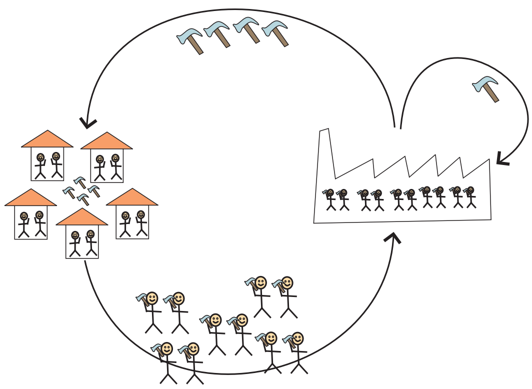
(d) What assumption about savings does Solow add to the assumptions about the production function set out above? What does this imply about the development of capital and aggregate production? (Set population growth equal to zero.)

2.2 Assume an economy in which the representative firm has a Cobb–Douglas production function,

$$F(K,L,A_L) = (A_L L)^{1-\alpha} K^{\alpha}.$$

Firms are price takers.

(a) The economy is illustrated below, and is in a long-run steady state with constant A_L. Find the values of A, δ, and s, where A is total factor productivity, i.e. $A = A_L^{1-\alpha}$.



(b) Derive the elasticity of substitution between the inputs.

(c) Explain what this implies about the factor shares of the inputs.

(d) Explain what this implies about the direction of technological change.

2.3 (a) What drives long-run growth according to the Solow model?

(b) What is the key unanswered question of the Solow model?

(c) When might the assumption of a single product be particularly problematic when using the model to explain and predict?

3. DHSS

3.1 Assume you own a quantity of a resource S, which you can extract at zero cost at any time. Furthermore, you know with certainty both the price path of the resource on the market, and the interest rate. You are a price taker.

(a) Explain intuitively your decision rule for when to extract and sell the resource.

(b) Derive the rule mathematically by setting up a Lagrangian function.

(c) This simple scenario suggests that market prices of non-renewable resources should rise rather rapidly. Explain how the conclusion above concerning the behaviour of an individual resource holder leads to the prediction of rising market price.

3.2 Assume an economy described by the following equations.

$$\begin{aligned} Y &= (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta} \\ \dot{A}/A &= g_A \\ \dot{L}/L &= n \\ \dot{K} &= Y - C - \delta K \\ S &\geq \int_0^{\infty} R_t \,\mathrm{d}t. \end{aligned}$$

Discuss possible interpretations of the equations, focusing especially on the role of resources in the economy.

3.3 Assume an economy described by the following equations.

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^{\alpha} R^{\beta} \\ \dot{K} &= Y - C \\ S &= \int_0^{\infty} R_t \,\mathrm{d}t. \end{aligned}$$

The usual interpretations apply. Assume that A_L is growing at rate g_A (i.e. $\dot{A}_L/A_L = g_A$), while L is constant.

(a) Show that for any initial level of resource use R_0 we can always find a fixed rate of decay in resource consumption θ such that initial resource stock S is asymptotically exhausted. What is the relationship between S, R_0, and θ?

(b) Assume we have a balanced growth path on which Y grows at a constant rate g_Y, while a constant proportion s is invested, and K grows at a constant rate g_K. Demonstrate that Y/K must be constant.

(c) Differentiate the production function w.r.t. time and use $g_Y = g_K$ to find an expression for g_Y in balanced growth. Explain.

3.4 Assume an economy described by the following equations:

$$\begin{aligned} Y &= (A_L L)^{1-\alpha-\beta} K^{\alpha} R^{\beta} \\ \dot{A}_L/A_L &= g_A \\ \dot{L}/L &= n \\ \dot{K} &= sY - \delta K. \end{aligned}$$

Thus we have a standard set-up with population growth and depreciation, but R is now interpreted as land, and is simply a constant.

(a) Use the capital accumulation equation to help explain why Y/K must be constant on a balanced growth path.

(b) Differentiate the production function with respect to time, and then use the result that Y and K grow at equal rates on a b.g.p. to solve for g_Y and g_y on a b.g.p., where $y = Y/L$.

(c) Now assume that population actually rises *endogenously* as a function of total output Y, such that output per capita y is constant. Characterize the balanced growth path in this economy.

(d) We endogenized n. What else could we endogenize?

(e) Assume population growth stops in the model, that is we have $n = 0$ exogenously determined. (Why might this happen?) What is the new long-run growth path?

(f) Discuss what we can learn from the model about prospects for long-run growth under environmental constraints.

3.5 Assume an economy described by the following equations:

$$Y = \min\{A_L L, A_R R\}; \tag{1}$$

$$\dot{A}_L/A_L = g; \tag{2}$$

$$S_0 \geq \int_0^{\infty} R_t \,\mathrm{d}t. \tag{3}$$

Furthermore, we have that $A_{L0} L < A_R S_0$. The resource is free to extract, and open access (and cannot be stored after extraction).

(a) Interpret these equations.

(b) Characterize the development of this economy, assuming first that A_R is constant, then assuming that A_R starts equal to 1, then grows by 0.05 every year.

(c) What is wrong with these assumptions, according to critics of the 'Limits' approach such as Solow (*Is the end of the world at hand?*, Challenge, 1973) and Sagoff (*Carrying capacity and ecological economics*, Bioscience, 1995)?

3.6 Sagoff, *Carrying capacity and ecological economics*, p. 611:

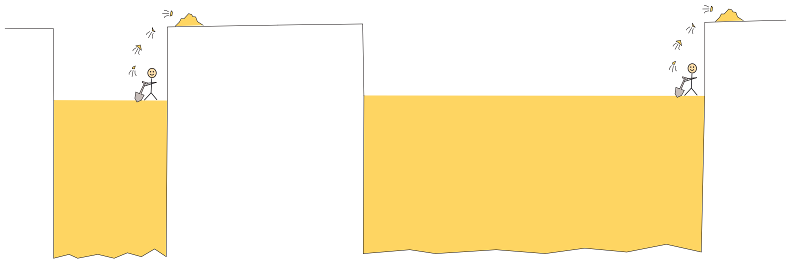
> Mainstream economists offer at least three arguments to show that knowledge and ingenuity are likely always to alleviate resource shortages.

The arguments are that

- Reserves are functions of technology,
- Technological progress allows us to substitute for scarce resources, and
- Technological progress increases the resource-efficiency of production.

Have the 'mainstream economists' to whom Sagoff refers—such as Dasgupta, Heal, Solow, and Stiglitz—backed up these assertions with testable (and tested) economic models?

3.7 With help from the picture, describe what factors may cause extraction costs to rise or fall over time. Can you make any predictions?



3.8 Consider the following economy. There is a constant population and a constant interest rate r. The aggregate production function is Cobb–Douglas in labour and the resource:

$$y = (a_y l_y)^{1-\alpha} x^{\alpha}.$$

Here α is a parameter between 0 and 1, a_y is labour productivity in final-good production, and l_y and x are the respective quantities of labour and resources used in production. The resource flow x is given by the following extraction function:

$$x = l_x a_{xt} / b_{xt}.$$

Labour inputs in extraction are l_x, the productivity of that labour is a_x, and b_x is an inverse productivity factor b_x representing the difficulty of extracting the resource: the *depth* of the marginal resource. The productivity indices a_y and a_x grow exogenously, and total labour L is fixed.

$$\begin{aligned} \dot{a}_y / a_y &= \theta_{ay} \\ \dot{a}_x / a_x &= \theta_{ax} \\ L &= l_x + l_y. \end{aligned}$$

We assume for simplicity that $\theta_{ax} = \theta_{ay}$. Finally, we assume that all markets are perfect, and we have a unit continuum of resource owners each with identical inhomogeneous endowments.

(a) Find an expression for the resource price p_x by taking the first-order condition on the final-good producer's profit function.

(b) Assume a b.g.p. on which quantities of labour are constant and depth b_x grows at a constant rate. Find expressions for the growth rate of x, p_x, and y on the b.g.p.

(c) Assume a primitive economy in which resource extraction is just beginning. What can we say about b_x. Characterize the b.g.p.! What happens over time?

(d) Assume a 'mature' b.g.p. on which b_x grows at a constant strictly positive rate θ_{bx}. You characterized this b.g.p. in part (b). Illustrate the transition from the primitive to the mature b.g.p. in a graph, in the case when $\theta_{bx} = \theta_{ax}$.

(e) Characterize the development of the economy if the resource is close to exhaustion, and there are no substitutes.

4. DTC and structural change

4.1 (a) Write down a CES production function for a firm using two inputs, labour L and a resource R, and associated levels of *input-augmenting knowledge* A_L and A_R. Can you say anything about parameter values?

(b) Assume that the units of final-good production are widgets per day. Give units for labour, the resource, and the levels of input-augmenting knowledge.

(c) What are the *factor shares* of the firm?

4.2 Assume an economy on an island with a single product, houses. The production function is CES, with inputs of labour L and trees R, with factor-augmenting technology levels A_L and A_R. It can be written

$$Y = [\alpha(A_L L)^\varepsilon + \beta(A_R R)^\varepsilon]^{1/\varepsilon}.$$

Parameters α and β are both equal to 1, whereas $\varepsilon = -1$. There are 100 people on the island who all work in production, and 10 trees/week wash up on the shore. All markets are perfect. The price of houses is normalized to 1.

(a) Assume that the islanders have a technology called 'saws' which allows them to cut the trees into planks, which can then rapidly be made into houses (final product). This technology corresponds to $A_L = 1$, $A_R = 100$. What is the GDP per capita on the island?

(b) Now assume that the islanders obtain a technology called 'sawmills', corresponding to $A_L = 100$, $A_R = 100$. What is GDP per capita now?

(c) Calculate the *prices* and *relative factor shares* of labour and trees in (a) and (b) above.

(d) A new house-building technology emerges which allows houses of the same quality to be built using half the number of identical planks. What does this imply about changes in A_L and A_R? Prices and factor shares? Explain.

4.3 Assume a single firm making a good Y using inputs L and R bought on perfect markets. The firm's production function is

$$Y = [(A_L L)^\varepsilon + (A_R R)^\varepsilon]^{1/\varepsilon}.$$

The firm determines the productivities of the inputs through investment, as follows:

$$A_L = \zeta_l \mathbf{A}_L z_l^\phi \quad \text{and}$$
$$A_R = \zeta_r \mathbf{A}_R z_r^\phi,$$

where $\mathbf{A}_L$ and $\mathbf{A}_R$ are general knowledge stocks exogenous to the firm, z_l and z_r are investments, ζ_l and ζ_r are positive parameters, and ϕ is a parameter less than one.

(a) Set up the firm's profit-maximization problem at time t as a Lagrangian, assuming it is myopic (not accounting for knowledge spilling over into future periods).

(b) Take first-order conditions in A_L and A_R, and in investment and input quantities, to find relative investments at the optimum.

(c) Assume that the results translate into an aggregate picture in which investments in knowledge are in proportion to factor shares, and the elasticity of knowledge to investment is fixed in each sector. Discuss the properties of the economy in that case, comparing to the standard DHSS model.

4.4 Assume an economy in which two products, y_1 and y_2, are produced by representative firms according to the following production functions:

$$\begin{aligned} y_1 &= \min(A_l l_1, A_{r1} r_1); \\ y_2 &= \min(A_l l_2, A_{r2} r_2); \\ l_1 + l_2 &= L; \\ r_1 + r_2 &= R. \end{aligned}$$

The As are productivity levels, l_1 and l_2 are quantities of labour, and r_1 and r_2 are quantities of resource inputs. We are interested in the effect of an exogenous increase in A_{r1} on total resource use in the economy, R. To help us investigate, we take first-order conditions and perform various algebraic manipulations to derive

$$\eta_r = -\frac{r_1}{r_1 + r_2}\left[1 - \left(1 - \frac{A_{r1}}{A_{r2}}\right)\eta_l\right],$$

where η_r is the elasticity of total resource demand R to an increase in A_{r1}, and η_l is the elasticity of l_1 w.r.t. the change in A_{r1}.

(a) Given this result, discuss the size of the rebound effect in the following cases.

i. When $\eta_l = 0$.

ii. When $\eta_l > 0$ and $A_{r1} = A_{r2}$.

iii. When $\eta_l > 0$ and $A_{r1} > A_{r2}$.

iv. When $\eta_l > 0$ and $A_{r1} < A_{r2}$.

(b) What conclusions can we draw about rebound in real economies? Policy?

4.5 The IATA (an air transport lobby group) argues that the price elasticity of demand for air travel is approximately 1, and therefore argues that policies to manage air travel demand by raising prices are likely to fail.[1] They recommend instead action to reduce emissions through technology.

To analyse the question, set up a model in which the production function of the representative producer of air transport is

$$Y = \min\{A_L L, A_E E\},$$

where Y is air transport, L is labour, E is energy use, and A_L and A_E are productivities. Furthermore, assume that there is constant elasticity demand for air transport,

$$Y = \alpha p_y^{-\eta}.$$

Here p_y is the price of air transport, α is a parameter and η is the elasticity.

(a) Find total costs for the representative producer in terms of input prices, productivities, and total energy use. (*Hint. First find costs in terms of prices and quantities of labour and energy. Then substitute for L using the fact that $A_L L = A_E E$ when the firm is minimizing costs.*) Then find the price p_y, i.e. unit cost.

(b) Use the demand function, and the fact that $Y = A_E E$ in equilibrium, to find energy use E as a function of input prices and productivities.

(c) Use your expression for E to find expressions for the elasticity of E to changes in: (i) energy productivity A_E; and (ii) energy price w_E.

(d) Assume that $A_L = 1$ and $A_E = 5$, and that $w_L = w_E = 100$. Calculate the elasticities, and comment on the relative sizes of the effects of energy efficiency and fuel taxes.

(e) What likely effects can we add to the analysis if we assume that taxes and/or research subsidies remain in the long run?

4.6 Assume two resources, C and D, used in quantities C and D, and with associated stocks of factor-augmenting knowledge A_C and A_D. The *augmented* resources $A_C C$ and $A_D D$ are perfect substitutes for one another. Assume that the stocks of knowledge grow independently, and furthermore assume that it is easier to boost D-augmenting knowledge than it is to boost A-augmenting knowledge.

[1] See for instance
`http://www.iata.org/SiteCollectionDocuments/air_travel_demand_summary.pdf`.

(a) Write down knowledge production functions for A_C and A_D which have the property of independence.

(b) Knowledge stocks are equal at time zero, but resource D is slightly cheaper than resource C, and remains so for 5 20-year investment periods. Describe in words how the economy develops over those 5 periods.

(c) In period 6 the regulator in this economy realizes that consumption of D is seriously damaging the environment. Discuss regulatory options, paying particular attention to the case when imposing emissions pricing (through taxes or tradable permits) is difficult in practice.

4.7 Assume an economy in which total aggregate production is a function of labour-intensive and resource-intensive production, as follows:

$$Y = Y_L^{\alpha} Y_R^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_L = A_l L,$$

where k_l is labour-augmenting knowledge and L is labour. The resource-intensive good is produced according to the following production function, where C is a clean input and D is a dirty input, and ε is a parameter less than one:

$$Y_R = [(A_c C)^{\varepsilon} + (A_d D)^{\varepsilon}]^{1/\varepsilon}.$$

Finally, knowledge stocks grow together, exogenously:

$$A_l = A_c = A_d;$$
$$\dot{A}_l / A_l = \theta.$$

(a) Find the shares in total product of Y_L and Y_R.

(b) Find the shares in total product of L and R, where $R = C + D$.

(c) Derive separate expressions for the factor shares of C and D in production of Y_R, first in terms of quantities, then in terms of prices.

(d) Assume that initially only factor D is available, but that at some time T factor C also appears on the market. What happens?

(e) Compare this economy to the real global economy, as observed over the last 150 years. Give one or more examples of 'new' resources which have emerged, and compare their emergence in reality to the emergence of C in the model.

(f) Discuss what—if anything—we can learn from the model regarding policy to reduce carbon dioxide emissions from the burning of fossil fuels.

4.8 (a) Write down a production function which has the potential for DTC increasing resource efficiency. Explain.

(b) Write down a production function which has the potential for DTC and substitution between resource inputs. Explain.

(c) Write down production and utility functions in an economy with the potential for substitution away from product categories in which the production process is resource-intensive. Explain.

5. Pollution

5.1 We can model both natural resources and polluting emissions as inputs into the aggregate production function:

$$Y = (A_L L)^{1-\alpha-\beta} K^{\alpha} R^{\beta}.$$

However, when we look at data we see very different trends. For natural resource extraction the rate of extraction tends to follow the overall rate of production of goods in the economy. Whereas for polluting emissions the rate of emissions has typically tracked GDP up to a point, after which emissions fall rapidly.

What is the cause of the difference!

5.2 Assume an economy with competitive markets with a single final good produced in quantity Y using inputs of labour L and resources R. The production function is as follows:

$$Y = (A_L L)^{1-\alpha} R^{\alpha} (1 - \psi D),$$

where A_L is labour productivity and D is the flow of pollution (which does not accumulate), ψ is positive and α is close to zero (so the pollutant has a small factor share). A_L and L grow exogenously at constant rates. Resources R can be produced a combination of two inputs X_i where $i = 1, 2$. The inputs are perfect substitutes, and

$$R = \sum_i X_i.$$

The inputs differ in two respects. Firstly, the costs of extraction w_i differ. Costs are constant for each input, hence they have constant prices, but $w_2 = (1+\gamma)w_1$, where $\gamma > 0$. Secondly, input 1 leads to polluting emissions D, according to the following equation:

$$D = X_1,$$

whereas the more expensive input does not cause any emissions. Utility U is production Y minus total extraction costs, $\sum_i w_i X_i$.

(a) Find an expression for MC_1, the marginal social cost of using input X_1, in terms of exogenous factors and R.

(b) Find a corresponding expression for MC_2.

(c) What are the marginal social *benefits* of using the respective inputs? Comment briefly.

(d) Find a condition for $MC_1 = MC_2$ in terms of R and $A_L L$, and explain what it implies about the switch from input X_1 to X_2.

(e) Describe the path of economic development in this economy as A_L and L grow (starting from a low level), assuming that the economy is optimally regulated. Explain.

(f) Discuss briefly the likely development path if there are multiple inputs which are perfect substitutes in making R, and which differ both in price and the degree to which they add to pollution flows D.

(g) Discuss briefly the difference it would make to the development of the economy if the alternative inputs were imperfect substitutes.

6. Labour supply and sustainable development

6.1 Assume a population of identical households indexed by i where the utility of a given household is described by the following function:

$$u_i = c_i^{\alpha_1} r_i^{1-\alpha_1-\alpha_2} (c_i/\bar{c})^{\alpha_2}.$$

Here c_i is consumption, r_i is leisure, and $\bar{c}$ is average consumption across all households. Furthermore, α_1 and α_2 are parameters the sum of which is less than 1.

(a) In this economy one agent's choice to work hard and take little leisure time has negative external effects on all other agents. Explain.

(b) In laissez faire all agents supply more labour than they would in an optimally managed economy. Explain.

(c) How could a regulator correct the externality and ensure an optimal solution?

(d) Discuss the potential relevance of the model (and extensions) to real-world economic and environmental policy.