Department of Economics
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Examination

Sustainable Development, NA0167.

Rules

Permitted aids: Pen and paper.

Answer 3 questions in total, out of 4 available. Each question is worth 20 points, and where a question is divided into parts, each part gives equal points. (If you answer 4, I will add up all your points and then multiply by 3/4.) As a broad guideline, there is one question related to each of the following topics.

1. Neoclassical growth theory, and the DHSS model.
2. Resource prices and quantities in neoclassical theory.
3. Directed technological change and sustainability.
4. Consumption, rebound, and sustainability.

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1. Consider the neoclassical growth model with zero population growth, and add the need for land in the production function. The quantity of land is simply fixed at R, and there is exogenous labour-augmenting technological progress.

$$Y = (AL)^{1-\alpha-\beta} K^{\alpha} R^{\beta}$$
$$\dot{A}/A = g_A$$
$$\dot{K} = sY - \delta K.$$

   (a) Interpret these equations.

   (b) Assume that there exists a balanced growth path (b.g.p.) on which all variables grow at constant rates. Find the growth rate of Y on this b.g.p.

   (c) Find the growth rates of the prices and factor shares of labour, capital, and land on the b.g.p.

   (d) Discuss the extent to which this model might give a useful —although highly simplified—picture of economic growth and resource use in the very long run.

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2. Assume that there is—in the global economy—a unit continuum of resource owners, who each have the option to hire labour (price w) and use it (alone) to extract their resource and sell it. The extraction function is

$$x_t = A_{xt} l_{xt} / B_{xt},$$

where x_t is the rate of extraction, A_{xt} is the productivity of the extraction input, l_{xt} is the quantity of the extraction input used, and B_{xt} is the depth of the resource. Units:

w_t	USD · worker^{-1} · year^{-1}
x_t	tons · year^{-1}
A_{xt}	tons · year^{-1} · worker^{-1}
l_{xt}	workers
B_{xt}	depth relative to baseline, hence unitless.

Each resource owner has an initial known stock of resource, S tons.

   (a) Write down an expression for the total cost of extraction for one resource owner (in USD/year), and use it to help you find an expression for the *unit* extraction cost (in USD/ton).

   (b) Assume the following: that labour productivity in the overall economy rises at a constant rate θ_y percent per year; that A_{xt} rises at a constant rate θ_x percent per year; and that there is a constant interest rate r percent per year. Use the expression you derived above to predict the trend in the resource price w_x in this economy in the following cases: (i) stocks S are large and homogeneous, and the date of exhaustion is very remote; (ii) stocks S are large and *inhomogeneous*, depth B_{xt} is increasing over time at a constant rate θ_b percent per year, and the date of exhaustion is very remote; (iii) the date of exhaustion is close.

(c) Choose a real resource, and discuss—in the light of the model—the long-run path for the price of that resource. (You should both discuss historical evidence and make predictions.)

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3. [A]s the earth's supply of particular natural resources nears exhaustion, and as natural resources become more and more valuable, the motive to economize those natural resources should become as strong as the motive to economize labor. The productivity of resources should rise faster than now—it is hard to imagine otherwise.

[Solow, *Is the end of the world at hand?*, Challenge, 1973, p47.]

(a) Over the last 300 years the price of energy has been falling compared to the price of labour. Has this led to a failure of energy-augmenting knowledge to grow as fast as labour-augmenting knowledge? Discuss evidence.

(b) Over the next 50 years the price of energy may well rise relative to the price of labour. Will this lead to rapid increases in the efficiency of energy use in sectors such as lighting and transport? Discuss theory and evidence.

Turn over

4. Assume an economy in which total aggregate production is a function of labour-intensive and resource-intensive production, as follows:

$$Y = Y_L^{\alpha} Y_R^{1-\alpha}.$$

The labour-intensive good is produced according to the following production function:

$$Y_L = k_l L,$$

where k_l is labour-augmenting knowledge and L is labour. The resource-intensive good is produced according to the following production function, where C is a clean input and D is a dirty input, and ϵ is a positive parameter less than one:

$$Y_R = [(k_c C)^{\epsilon} + (k_d D)^{\epsilon}]^{1/\epsilon}.$$

All goods earn their marginal product.

(a) i. Find the shares in total product of Y_L and Y_R.

ii. Find the shares in total product of L and R, where $R = C + D$. (That is, find $w_l L$ and $w_c C + w_d D$.)

iii. Find the relative factor shares of C and D (i.e. $w_c C/(w_d D)$) in terms of quantities of C and D on the market and the knowledge stocks.

Now assume that knowledge stocks k_c and k_d grow independently of one another, and that the growth rate of a given knowledge stock is proportional to the factor share of the relevant input.

(b) Assume that the factors are available in unlimited quantities at fixed exogenous prices, and that at time t, factor D is dominant, that is $w_d D \gg w_c C$.

i. Briefly describe the development of the economy from time t given laissez faire.

ii. Discuss options for a regulator who wishes to raise C at the expense of D.

(c) Discuss the extent to which this model—or adaptations of it—can help us to design policy to reduce carbon dioxide emissions from the burning of fossil fuels.