

A diagram illustrating sustainable development. It features a brown, jagged ground line. On the left, a yellow, teardrop-shaped cloud hangs over a brown mountain peak. In the center, a small orange stick figure stands on the ground. On the right, a green tree with a brown trunk stands on the ground. The text "Sustainable Development" is written in white on a brown background at the bottom of the diagram.

Sustainable Development

A diagram illustrating sustainable development, identical to the one on the left. It features a brown, jagged ground line. On the left, a yellow, teardrop-shaped cloud hangs over a brown mountain peak. In the center, a small orange stick figure stands on the ground. On the right, a green tree with a brown trunk stands on the ground. The text "Sustainable Development" is written in white on a brown background at the bottom of the diagram.

Sustainable Development

Part 7

Substitution between alternative resource inputs

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Recall:
$$Y = (A_L L)^{1-\alpha} (A_R R)^\alpha.$$

$$\max \pi = p_y (A_L L)^{1-\alpha} (A_R R)^\alpha - w_l L - w_r R;$$

$$w_r R = \alpha Y.$$

Now
$$R_t = [(\gamma_c A_{ct} X_{ct})^\epsilon + (\gamma_d A_{dt} X_{dt})^\epsilon]^{1/\epsilon}.$$

Assume $A_c = A_d = A$, and fix $A_R = 1$.

$$Y_t = (A_t L_t)^{1-\alpha} R_t^\alpha,$$

$$R_t = A_t [(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon]^{1/\epsilon},$$

and
$$C_t = Y_t - (w_{ct} X_{ct} + w_{dt} X_{dt}),$$

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$$\pi = w_{rt}A_t [(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon]^{1/\epsilon} - w_{ct}X_{ct} - w_{dt}X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c/w_c)^{\epsilon/(1-\epsilon)}$$

and $w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}.$

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$$\pi = w_{rt}A_t [(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon]^{1/\epsilon} - w_{ct}X_{ct} - w_{dt}X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c/w_c)^{\epsilon/(1-\epsilon)}$$

and $w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}.$

So the relative factor shares of the resources are

$$\frac{w_c X_c}{w_d X_d} = \left(\frac{\gamma_c/w_c}{\gamma_d/w_d} \right)^{\epsilon/(1-\epsilon)}.$$

This implies that the resource that is cheaper per efficiency unit takes the larger factor share, and the advantage is bigger the higher is the substitutability between the resources (i.e. when $\epsilon \rightarrow 1$).

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$$\pi = w_{rt}A_t [(\gamma_c X_{ct})^\epsilon + (\gamma_d X_{dt})^\epsilon]^{1/\epsilon} - w_{ct}X_{ct} - w_{dt}X_{dt},$$

$$w_c X_c = w_r (R/A)^{1-\epsilon} (\gamma_c X_c)^\epsilon$$

and $w_d X_d = w_r (R/A)^{1-\epsilon} (\gamma_d X_d)^\epsilon,$

$$w_c X_c = w_r^{1/(1-\epsilon)} (R/A) (\gamma_c/w_c)^{\epsilon/(1-\epsilon)}$$

and $w_d X_d = w_r^{1/(1-\epsilon)} (R/A) (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}.$

Because we have perfect markets, price equals unit cost so

$$\begin{aligned} w_r &= (w_c X_c + w_d X_d) / R \\ &= w_r^{1/(1-\epsilon)} (1/A) \left[(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)} \right] \\ &= \left\{ A / \left[(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)} \right] \right\}^{\epsilon/(1-\epsilon)}. \end{aligned}$$

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So we have

$$w_r = \left\{ A / [(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{\epsilon/(1-\epsilon)} .$$

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So we have

$$w_r = \left\{ A / [(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{\epsilon/(1-\epsilon)}.$$

And since $w_r R = \alpha Y$ we have

$$w_r = \alpha (AL/R)^{1-\alpha},$$

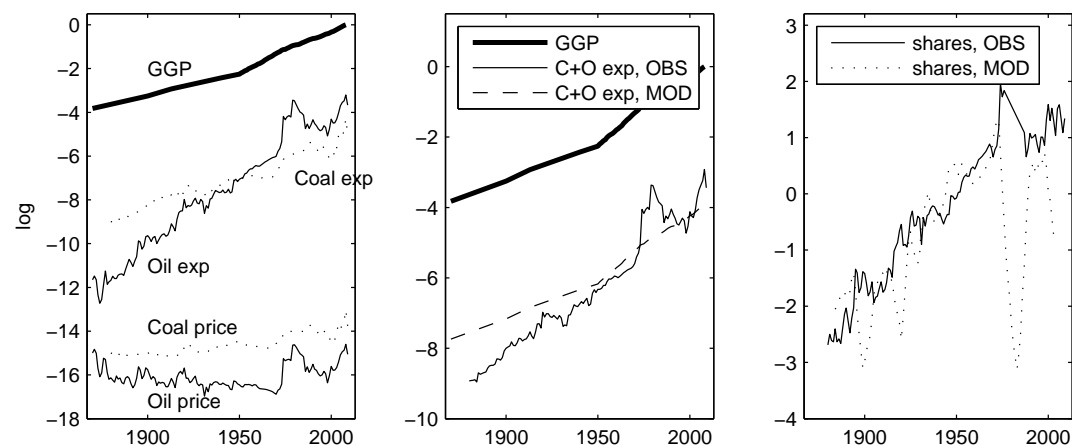
and we can eliminate w_r to yield

$$R = AL \left\{ \alpha [(\gamma_c/w_c)^{\epsilon/(1-\epsilon)} + (\gamma_d/w_d)^{\epsilon/(1-\epsilon)}] \right\}^{1/(1-\alpha)}.$$

So if w_c and w_d are both constant then R grows at the same rate as Y , i.e. $g + n$, the sum of the growth rates of labour productivity and population.

A simple model with alternative resource inputs

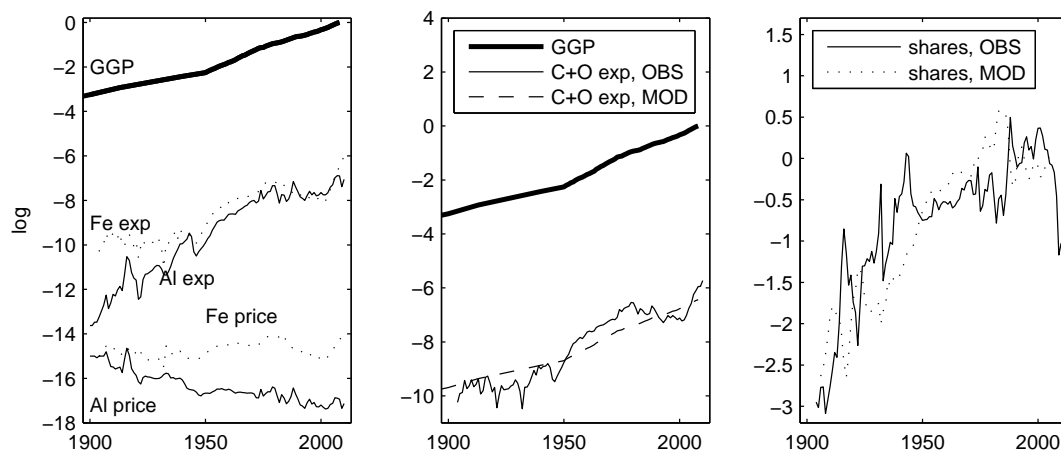
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Long-run growth in prices and factor expenditure, compared to growth in global product, for crude oil and coal, and a test of the model. In the left-hand figure we see observed prices and expenditures, with expenditures compared to global product. In the middle figure we see observed total expenditure on coal and oil, compared to the model prediction (based on the prices). And in the right-hand figure we see the observed relative factor shares of coal and oil, compared to the model prediction. In the calibrated model we have $\alpha = 0.02$, $\gamma_c/\gamma_d = 0.55$, and $\epsilon = 0.76$.

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Long-run growth in prices and factor expenditure, compared to growth in global product, for iron and aluminium, and a test of the model. In the left-hand figure we see observed prices and expenditures, with expenditures compared to global product. In the middle figure we see observed total expenditure on iron and aluminium, compared to the model prediction (based on the prices). And in the right-hand figure we see the observed relative factor shares of iron and aluminium, compared to the model prediction. In the calibrated model we have $\alpha = 0.002$, $\gamma_c/\gamma_d = 50$, and $\epsilon = 0.55$.

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Recall that relative investments are equal to relative factor shares in a model with L and R :

$$\frac{z_{lt}}{z_{rt}} = \frac{w_{lt}L_t}{w_{rt}R_t} = \left(\frac{A_{lt}L_t}{A_{rt}R_t} \right)^\epsilon.$$

In a model with C and D making R we have

$$\frac{z_{ct}}{z_{dt}} = \frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}C_t}{A_{dt}D_t} \right)^\epsilon.$$

If we add the assumption that knowledge stocks grow independently then we have

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{z_{ct}}{z_{dt}} \right)^\phi \left(\frac{\zeta_d}{\zeta_c} \right).$$

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$$\frac{z_{ct}}{z_{dt}} = \frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}C_t}{A_{dt}D_t} \right)^\epsilon.$$

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{z_{ct}}{z_{dt}} \right)^\phi \left(\frac{\zeta_d}{\zeta_c} \right).$$

Now assume a b.g.p. on which relative prices are exogenous and constant. Then z_c/z_d must be constant, and also A_c/A_d . So

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = 1 = \left(\frac{w_{ct}C_t}{w_{dt}D_t} \right)^\phi \left(\frac{\zeta_d}{\zeta_c} \right) = \left(\frac{A_{ct}C_t}{A_{dt}D_t} \right)^{\epsilon\phi} \frac{\zeta_d}{\zeta_c}.$$

So on a b.g.p. the shares of C and D are fixed. But is the b.g.p. stable?

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Focus on intuition.

Imagine the economy is on a b.g.p., and then a small shock shifts it such that the share of C increases. What happens?

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We have
$$\frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}C_t}{A_{dt}D_t} \right)^\epsilon$$

and
$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{w_{ct}C_t}{w_{dt}D_t} \right)^\phi,$$

hence
$$\frac{w_{ct}C_t}{w_{dt}D_t} = \left(\frac{A_{ct}}{A_{dt}} \right)^{\epsilon/(1-\epsilon)} \left(\frac{w_{ct}}{w_{dt}} \right)^{-\epsilon/(1-\epsilon)}$$

and
$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct}/w_{ct}}{A_{dt}/w_{dt}} \right)^{\epsilon\phi/(1-\epsilon)}.$$

Multiply both sides by $\left(\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} \right)^{-\epsilon\phi/(1-\epsilon)}$ to obtain

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct-1}/w_{ct}}{A_{dt-1}/w_{dt}} \right)^{\epsilon\phi/(1-\epsilon(1+\phi))}.$$

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We have

$$\frac{A_{ct}/A_{ct-1}}{A_{dt}/A_{dt-1}} = \left(\frac{A_{ct-1}/w_{ct}}{A_{dt-1}/w_{dt}} \right)^{\epsilon\phi/(1-\epsilon(1+\phi))} .$$

Assume we are on a b.g.p., and let A_c rise a little due to a shock. What happens?

Is the b.g.p. stable?

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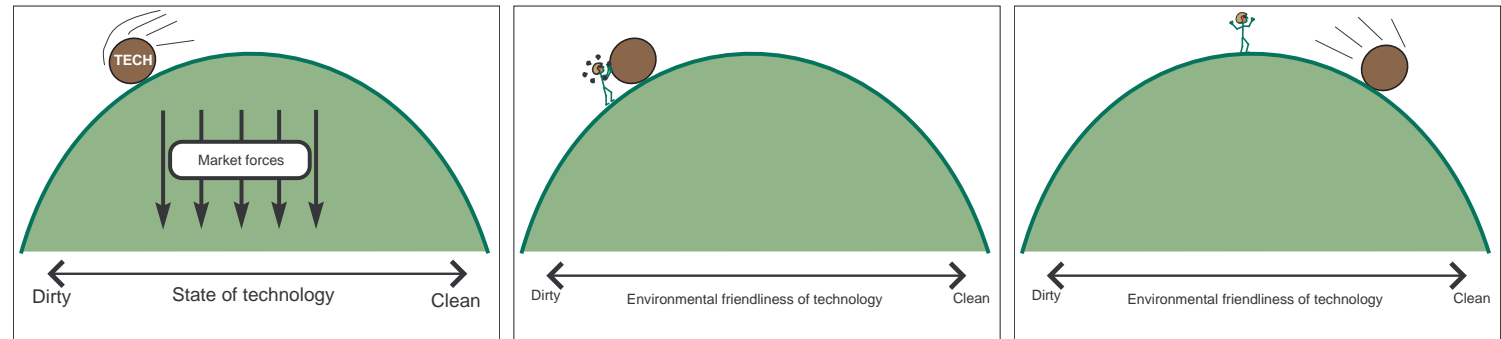


Figure 1: Illustration of how relative prices (the shape of the economic landscape) determine the relative levels of technology augmenting clean and dirty inputs in the model, and the role of a regulator.

Evidence?

What went wrong?