Chapter 2

Malthusian growth

Technological progress and Malthusian growth

Technological progress and Malthusian growth

$$Y = (A_L L)^{1-\alpha}(A_R R)^{\alpha},$$

$$Y/L = A_L^{1-\alpha}(A_R R/L)^{\alpha},$$

$$L_{t+1} - L_t = \theta L_t(1 - L_t/L_t^*),$$

$$\bar{y} = A_L^{1-\alpha}(A_R R/L^*)^{\alpha},$$

$$L^* = \frac{A_R R}{\bar{y}^{1/\alpha}} A_L^{(1-\alpha)/\alpha},$$

$$A_R = 1, \quad A_{Lt}/A_{Lt-1} = 1 - \delta + \zeta(\Omega L_{t-1})^{\phi}.$$

population L,
basic need $\bar{y}$
Quality A_R,
fixed.
Area R
Invention
Carrying capacity L^*

Technological progress and Malthusian growth
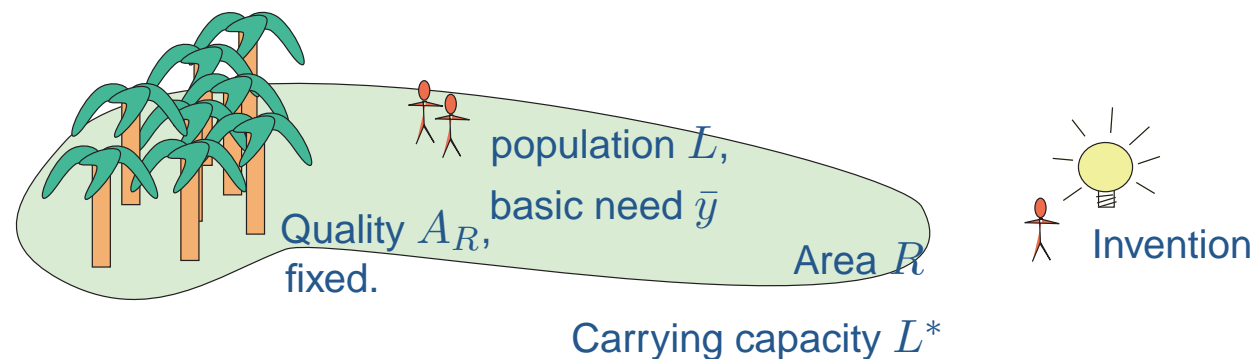
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$$\alpha = 0.4 \qquad \zeta = 0.020012$$

$$\phi = 0.2 \qquad \theta = 0.06$$

$$\delta = 0.98 \qquad \Omega L(0) = 1.$$

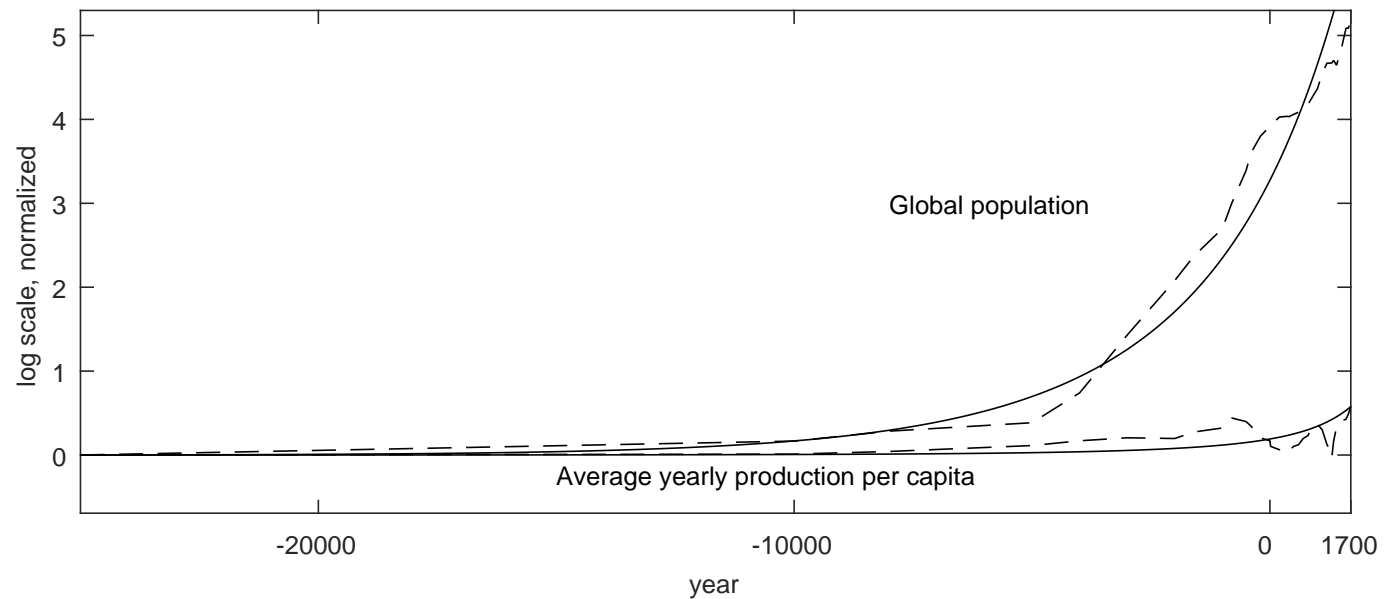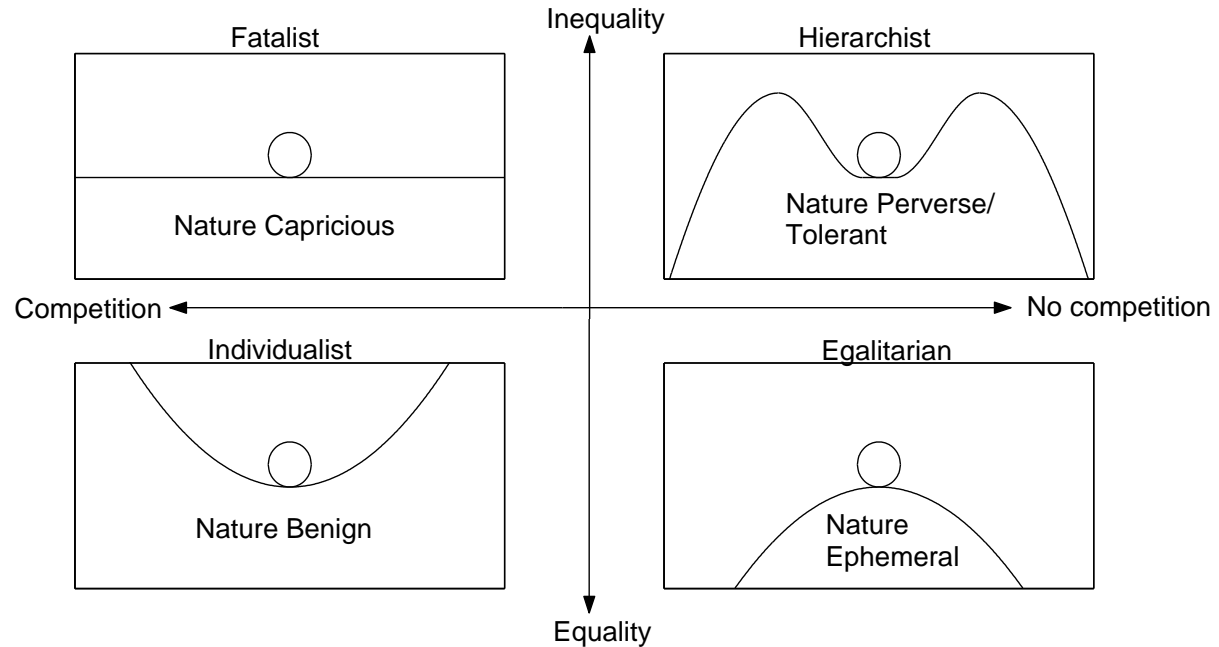# Technological progress and Malthusian growth

Figure 1: Global product and population. Continuous lines, model simulation; dashed lines, historical data from Brad DeLong.

A fragile ecosystem and megafaunal extinction

A fragile ecosystem and megafaunal extinction

States of nature, adapted from Thompson et al. 1990

We choose the hierarchist's approach of trying to explain
megafaunal extinction as the regrettable result of bad management
of the 'resource'.

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Since the extinctions occurred over relatively short timescales compared to preindustrial rates of technological progress, we can safely ignore technological progress in our model.

The simplest possible approach is to build a predator–prey model in which humans are completely dependent on 'harvesting' megafauna, the population of which decline under harvesting pressure.

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Y is harvest, L human population, N animal population. Linear harvest fn, logistic human population growth:

$$Y = \phi L N.$$

$$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(Y_t/L_t)].$$

When $Y/L = \bar{y}$, human popn is stable. Insert $Y/L = \phi N$ to yield

$$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(\phi N_t)]. \tag{1}$$

This is the equation for the dynamics of human population given the animal population. When $N = \bar{y}/\phi$, human popn stable.

Logistic animal population growth, with harvest pressure:

$$N_{t+1} - N_t = \theta_N N_t [1 - N_t/N^*] - \phi L_t N_t. \tag{2}$$

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Figure 2: Development of the numerical model over time

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What is the maximum sustainable human population?

Why don't we get there in the simulation?

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The model doesn't lead to extinction. Why not?

How could we adapt the model to deliver extinction?

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Add small animals S:

$$Y = L(\phi_N N + \phi_S S),$$

Stock S adapts instantly to hunting pressure, hence $S = S^* - \phi_S SL$ and $S = S^*/(1 + \phi_S L)$. Then

$$Y/L = \phi_N N + \phi_S S^*/(1 + \phi_S L),$$

and

$$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(\phi_N N_t + \phi_S S^*/(1 + \phi_S L_t))].$$

Recall

$$N_{t+1} - N_t = \theta_N N_t (1 - N_t/N^*) - \phi_N L_t N_t.$$

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$$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(\phi_N N_t + \phi_S S^*/(1 + \phi_S L_t))].$$
$$N_{t+1} - N_t = \theta_N N_t (1 - N_t/N^*) - \phi_N L_t N_t.$$

Assume $N = 0$. Then we have $Y = L\phi_S S$,
$Y/L = \phi_S S^*/(1 + \phi_S L)$, and
$L_{t+1} - L_t = \theta_L L_t [1 - \bar{y}/(\phi_S S^*/(1 + \phi_S L_t))]$. Solve for L_t
when $L_{t+1} - L_t = 0$ to yield

$$L = S^*/\bar{y} - 1/\phi_S.$$

This shows that if $\phi_S S^* < \bar{y}$ then the small animals are too scarce
(or hard to catch) to sustain a human population on their own.

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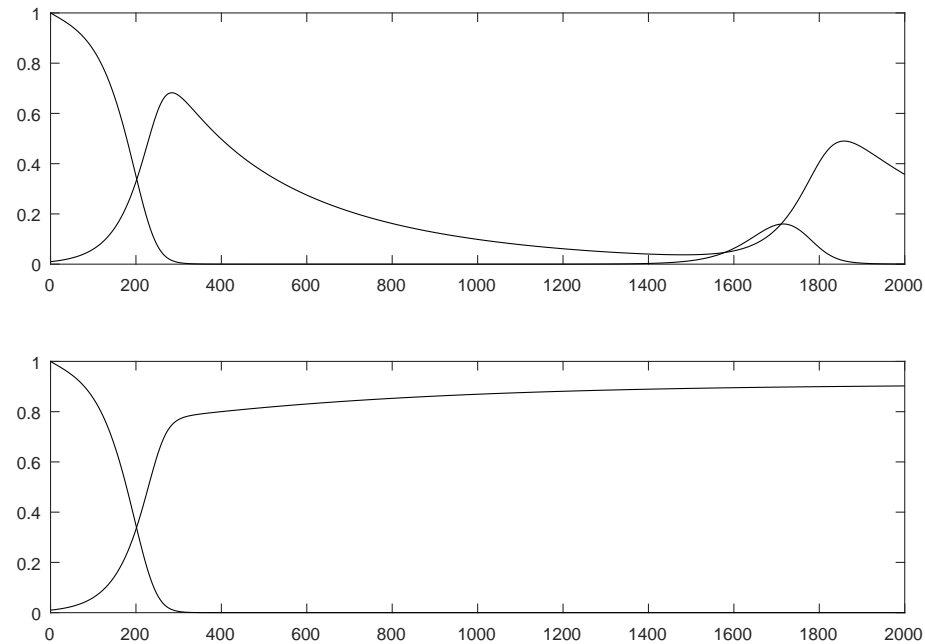
Figure 3: Development of the numerical model over time. Top, $\phi_S S^* < \bar{y}$; bottom, $\phi_S S^* > \bar{y}$.

What is going on here?

The post-Malthusian economy

- Technological progress and Malthusian growth
- A fragile ecosystem and megafaunal extinction

The post-Malthusian economy

Why are Malthusian models not relevant to modern economies?

At least, not unless we assume cataclysmic environmental damages.

What kind of models are relevant?